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ABSTRACT

This guide is designed to help classroom teachers implement the Alabama Course of Study: Mathematics K-12. It is inclusive also of the objectives tested by the Stanford Achievement Tests and the Alabama Basic Competency Tests. One characteristic of the curriculum guide is that it clearly states what students should learn in each grade level. These objectives are called student outcomes and appear in the table of contents. The majority of the material, however, consists of classroom activities that lead students to the desired outcomes. The curriculum guide specifies what to teach as well as ways to teach it. Each section of the guide contains a specific table of contents, which lists what students should learn; activities; original sources; and student outcomes. Activities are preceded by a description of the area of mathematics covered, the specific student outcomes that are addressed in the activities, and notes for the teacher, which convey information important for effectively carrying out the activities. Topics include symbols, numerals, signed numbers, open phrases, open sentences, solution sets, postulates, properties of operations, monomials, polynomials, equations, inequalities containing one and two variables, word problems, special products, factoring, operations with fractions, first degree equations, real numbers, and quadratic equations. (KR)

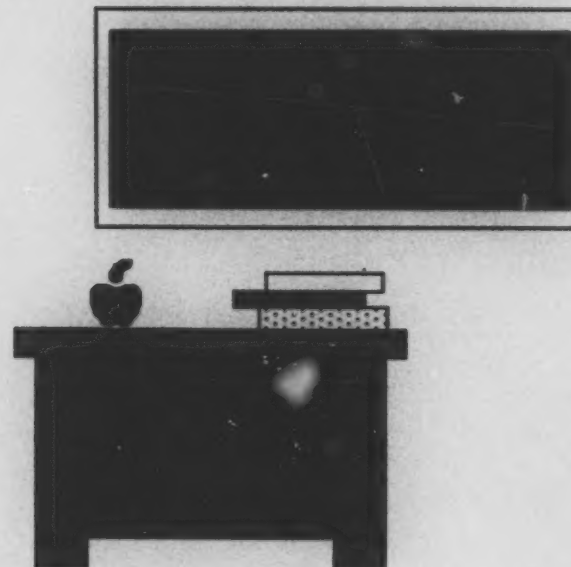
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MATHEMATICS

CURRICULUM GUIDE ALGEBRA I



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Bulletin 1987, No. 48

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MATHEMATICS CURRICULUM GUIDE ALGEBRA I

ALABAMA STATE DEPARTMENT OF EDUCATION

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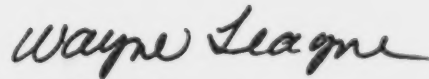
Project Director

FOREWORD

This Mathematics Curriculum Guide: Algebra I is designed to help classroom teachers implement the Alabama Course of Study: Mathematics K-12 (Bulletin 1984, No. 16). It is inclusive also of the objectives tested by the Stanford Achievement Tests.

One characteristic of the curriculum guide is that it clearly states what students should learn in Algebra I. These are called STUDENT OUTCOMES and appear in the Table of Contents. The majority of the material, however, consists of classroom ACTIVITIES that lead students to the desired outcomes. The curriculum guide, then, specifies what to teach as well as ways to teach it.

The Mathematics Curriculum Guide: Algebra I is supplemental in nature. It will add to material found in textbooks and complement locally developed skills continuums, scope and sequences, and/or curriculum guides. In the past, we have been most successful when we clearly defined what students should learn, gave examples of how it can be taught, and then tested for mastery. This curriculum guide is another step in that direction.



Wayne Teague
State Superintendent of Education

INTRODUCTION

The Mathematics Curriculum Guide: Algebra I is one of a series of curriculum guides to be developed by the Alabama State Department of Education. Its production is part of a larger plan to provide LEAs with a curriculum guide in every subject area K-12, immediately following the Course of Study adoption in the same subject area. The Mathematics Curriculum Guide K-6 is packaged separately. Each of the other secondary mathematics guides (Math - Grades 7 and 8, General Math, Consumer Math, Pre-Algebra, Geometry, Algebra II, Trigonometry/Analysis, and Applied Vocational Math will be packaged separately also and will be published shortly.

The Algebra I guide has three distinct sections. It begins with a TABLE OF CONTENTS which lists what students should learn (student outcomes) in Algebra I. After each outcome, a page number(s) directs the reader to classroom activities that address the specific outcome.

The second and largest section of the guide consists of classroom ACTIVITIES. Activities are preceded by a description of the area of Algebra covered, the specific student outcomes that are addressed in the activities, and Notes for the Teacher that convey information important for effectively carrying out the activities.

The third section is referred to as the ORIGINAL SOURCES AND STUDENT OUTCOMES portion of the guide. It summarizes the relationship of student outcomes to the original sources from which the outcomes were taken. (The original sources for the Mathematics Curriculum Guide: Algebra I outcomes were the Alabama Course of Study and the Stanford Achievement Test objectives.) This section of the guide will serve as a reference for local school systems that may wish to trace the source of each outcome or to rate each outcome as being basic, standard, or advanced.

The use of the Mathematics Curriculum Guide: Algebra I is not mandatory. Instead, it is offered as material that will help implement the Mathematics Course of Study and prepare students for the Stanford Achievement Tests. The Mathematics Curriculum Guide: Algebra I can be used as it is in LEAs that currently have no guide, or it can serve as a starting point for developing a similar guide locally. In either case, the involvement of local educators in the development of curricula is valued and encouraged. It is hoped that this guide will bring about local expansion activities such as writing outcomes for "optional" mathematics courses; adding to the list of outcomes for a given subject; writing additional activities in specific areas of identified weakness; rating outcomes as basic, standard, or advanced; and developing uniform end-of-unit tests.

ACKNOWLEDGEMENTS

From its inception, the development of curriculum guides has been viewed as a cooperative effort between specialists in the Division of Student Instructional Services and selected classroom teachers from the public schools of Alabama. The Curriculum Development Section, under the leadership of Katherine Mitchell, has the responsibility for coordinating this project.

Work on the Mathematics Curriculum Guide: Algebra I started in January 1986, when Linda G. Harris was selected as Project Director and the identification of student outcomes began. The second phase of the project took place during July and August when five classroom teachers joined State Department of Education specialists in the writing of activities that teach the identified outcomes. For twelve days the following teachers offered their best ideas:

James Davidson	- Jasper City Schools
Patricia Edwards	- Etowah County Schools
Myrtle Johnson	- Eufaula City Schools
Ann Lilly	- Tuscaloosa City Schools
Wanda Motes	- Enterprise City Schools

Over a period of eleven months, a team of State Department of Education specialists identified and reviewed student outcomes, wrote and rewrote activities, read activities for clarity, and proofed various drafts. Suzan Dunn and Robin Lanier contributed the graphics for the guide. Others who were directly involved with the Mathematics Curriculum Guide: Algebra I were Delton Blalock, Dorothy DeMars, Martha Jungwirth, Nell Kilpatrick, Linda Pledger, and Paul Wylie. Shirley Dorman was the primary typist for the document; Robin Lanier and Leigh Ann Lewis assisted her as needed. We appreciate the work of all these people and their co-workers, who made the adjustments and sacrifices necessary to carry out their own programs and, at the same time, made the Mathematics Curriculum Guide: Algebra I a division product.

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ALGEBRA 1

AREA: SYMBOLS, NUMBERS, AND NUMERALS

OUTCOME: 1. Use the vocabulary and symbols related to equations, inclusively, as they are introduced.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Know the meaning of the following: numerical expression, numeral, equality, inequality, variable, and variable expression.
2. Know the symbols for operations and their results: $+$, sum; $-$, difference; $\times$, product; and $\div$, quotient.

Steps to Success

1. A symbol sometimes represents a person, a thing, or an idea. In arithmetic symbols are used to represent numbers.
2. A number is an idea. It is something to think about. The symbol for a number is a numeral.
3. When writing mathematical sentences, symbols are used to relate to the reader the relationship of the numbers, quantities, or expressions.

ACTIVITY 1 - OUTCOME 1

Ask the students to draw on a piece of paper something that might represent them personally.

A cheerleader might draw a megaphone.

A football player might draw a football.

An artist might draw a palette.

Let several students share these drawings with the class.

Lead the discussion to the idea of "symbols" and the necessity for symbols in our society, particularly in mathematics.

ACTIVITY 1 - OUTCOME 1 (Cont'd)

Remind students: when two numbers or expressions are compared, there are three types of symbols to use for comparison purposes.

TYPE 1: SYMBOL OF EQUALITY

= means "equal" or "is equal to"

This symbol indicates that two numbers or expressions are the same. A statement of this kind is called an equality. Any statement of equality is called an equation.

Example 1: $2 \times 4 = 2 + 2 + 2 + 2$

Example 2: $10 \times 0 = 0 \times 5$

Example 3: $a + 2 = 6$

TYPE 2: SYMBOLS OF INEQUALITY

> means "is greater than"

< means "is less than"

≠ means "is not equal to"

These symbols indicate that two numbers or expressions do not have the same value. They are called inequalities.

Example 1: $14 - 3 = 15 + 2$

Example 2: $8 < 12$

Example 3: $17 > 10$

Example 4: $a + 2 < 6$

NOTE: For students having difficulty remembering which symbol represents "is less than" and which "is greater than," have them complete the "less than" or "greater than" symbol with a dashed circle to form a Pac Man. Remember, Pac Man goes for the larger score.

$2 \overset{\curvearrowright}{<} 4$
2 is less than 4

$\frac{1}{2} \overset{\curvearrowright}{>} \frac{1}{8}$

$\frac{1}{2}$ is greater than $\frac{1}{8}$

ACTIVITY 1 - OUTCOME 1 (Cont'd)

TYPE 3: COMBINATION OF EQUALITY OR INEQUALITY

$\leq$ means "is less than or equal to"

$\geq$ means "is greater than or equal to"

These symbols indicate that two numbers or expressions may or may not have the same value.

Example 1: $18 \leq 20$ either $18 < 20$ or $18 = 20$

Example 2: $a \geq 12$ either $a > 12$ or $a = 12$

ALGEBRA I

AREA: SYMBOLS, NUMBERS, AND NUMERALS

- OUTCOMES:
2. Graph real numbers on the number line.
 3. Compare numbers using the number line.
 4. Relate the absolute value of a number with its distance from the origin on the number line.
 5. Illustrate the operations of addition and subtraction by the use of arrows representing movement on the number line.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Know the meaning of the following: real numbers, absolute value, origin, number line, positive numbers, negative numbers, directed numbers, and coordinates of a point.

Steps to Success

1. Any number that is either a positive number, a negative number, or zero is a real number. On a number line, 0 is the origin or starting point. Any point to the left of zero (less than zero) is a negative direction. Any point to the right of zero (greater than zero) is a positive direction.
2. There is only one point paired with any real number on a number line.

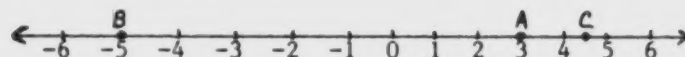
ACTIVITY 1 - OUTCOMES 2, 3, and 4

Ask two students to come to the front of the room. State, "I (the teacher) am the origin." Face the front of the room with your back to students. Ask first student (Suzy) to start where you are and take two steps to your right. Ask, "On a number line, Suzy's coordinate would be what?" (+2) Ask second student (Jim) to start at the same place and take three steps to your left. Ask, "On the number line, what is Jim's coordinate?" (-3)

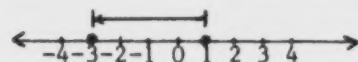
Discuss with students other examples of directed numbers, such as temperature scale on a thermometer, directions East - West or North - South, and bank deposits or withdrawals.

ACTIVITY 1 - OUTCOMES 2, 3, and 4

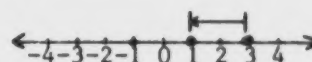
Example 1: Graph and label $A = 3$, $B = -5$, and $C = 4\frac{1}{2}$ on a number line.



Example 2: Show that $-3 < 1$ and $1 < 3$.



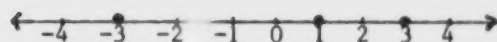
and



$$-3 < 1$$

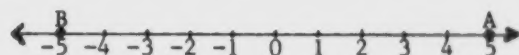
$$1 < 3$$

This can be read in combination as follows: -3 is less than 1 which is less than 3 .

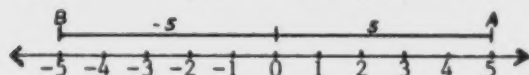


$$-3 < 1 < 3$$

Example 3: Graph $A = 5$ and $B = -5$.



NOTE: $|5| = 5$ and $|-5| = 5$

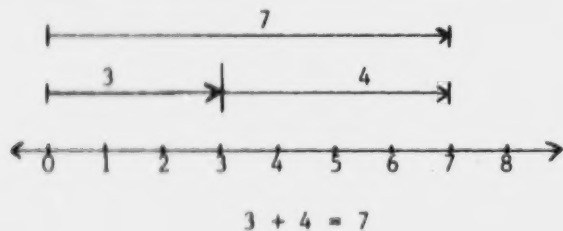


The absolute value of a number is its distance from zero without regard to the direction.

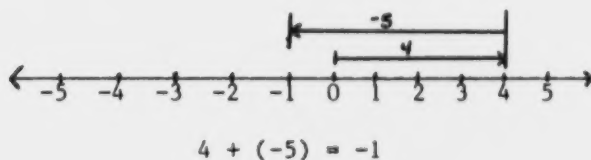
ACTIVITY 2 - OUTCOME 5

Ask two other students to come to the front of the room. Ask them to start at the origin and, side-by-side, walk three steps to the right. Ask one of the students to walk four additional steps.

Now ask the class to represent this action taken on a number line.



Now ask the two students to start at the origin again and walk four spaces in a positive direction. Ask one to stand still and the other to walk five spaces in a negative direction. Ask the class to represent the action taken on a number line.



ALGEBRA I

AREA: SYMBOLS, NUMBERS, AND NUMERALS

- OUTCOMES:
6. Associate a numeral with an equivalent expression of the same number.
 7. Simplify expressions containing more than one grouping symbol.
 8. Simplify expressions when grouping symbols are omitted.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Know the signs of operation: $+$, $-$, $\times$, $\div$.
2. Know the signs of grouping: $()$ parentheses, $—$ division bar, and $[]$ brackets.
3. Understand what is meant by substitution.

Steps to Success

1. A numerical expression involves numerals and signs of operation.
2. Sometimes a numeral is enclosed in parentheses to indicate it is a numeral.
3. A procedure must be followed when more than one operation is indicated. This procedure is called order of operations and is as follows:
 - (a) Perform all multiplications and divisions first, in order from left to right.
 - (b) Perform all additions and subtractions, in order from left to right.
4. To simplify a numerical expression which involves parentheses, perform the operation(s) in the parentheses first.
5. When there are two or more grouping symbols in one expression, perform the operation(s) in the innermost symbol first.
6. Changing the numeral by which a number is called does not change the value of the expression.

ACTIVITY 1: OUTCOME 6

TYPE 1: EQUIVALENT NUMERICAL EXPRESSIONS

Ask students what is meant by the word substitute in the following questions:

What is a substitute teacher?

What is a substitute basketball player?

Can you think of other substitutes?

Attention should be drawn to the idea of "stand-in" or "replacement."

Write a big "8" on the board. Ask for other ways of writing 8. ~~7777~~ ///; $4 + 4$; $10 - 2$; eight; or the number of sections in which a watermelon can be cut. Each of these is an equivalent expression of the numeral 8. The numeral 8 is a symbol which can be substituted for either of the ideas discussed.

What symbol may replace the question mark (?) in the following expressions to make an equivalent expression?

$$\begin{array}{ll} 1 + 2 = ? & (3) \\ 16 - 2 + 0 = ? & (14) \\ 8 \times 12 = ? & (96) \end{array}$$

TYPE 2: EQUIVALENT VARIABLE EXPRESSIONS



Draw a barrel on the board and label it A.

State, "The barrel is named A. It has many numerals in it. I am going to reach over and pick up one."
(Pantomime picking up a number in a closed fist so no one can see it.)

Ask, "Do any of you know what number this is?" Encourage someone to give his guess and say, "Yes, how did you do that? That was the number I picked out of the barrel." (For example, the student may have guessed 10.)

State, "We can call this value of "A" ten and write it in symbols as $A = 10$."

Pick other numbers until the students understand that a variable is a symbol which may represent a number.

ACTIVITY 1 - OUTCOME 6 (Cont'd)

Look at some examples of variable expressions and replace the variable with a number to make it a true statement.

Example 1: $28 = A - 19$
 $28 = \quad - 19$
 $28 = 47 - 19$

Example 2: $33 - N = 16$
 $33 - \quad = 16$
 $33 - 17 = 16$

ACTIVITY 2 - OUTCOMES 7 and 8

There are rules or directions in our society which help keep order and simplify things. For example, what happens when a traffic light goes out at a busy intersection? It could be chaotic. But if rules of courtesy and safe driving are followed, everyone will make it through the intersection safely.

The same is true for mathematics. There are sets of rules that must be followed. Two such sets of rules are the order of operations and the procedures for signs of grouping.

TYPE 1: ORDER OF OPERATION

Ask the students:

"Do any of you know my dear Aunt Sally?"

"My dear Aunt Sally can help you to remember in what order to perform operations when more than one operation is indicated."

My - multiplications
dear - divisions
Aunt - additions
Sally - subtractions

ACTIVITY 2 - OUTCOMES 7 and 8 (Cont'd)

Example: $2 \times 6 + 3 - 2 \div 2 =$

Perform all multiplications and divisions in order from left to right.

$$12 + 3 - 1 =$$

Perform all additions and subtractions in order from left to right.

$$\begin{array}{r} 15 - 1 = \\ 14 \end{array}$$

TYPE 2: SIGNS OF ORDER

Ask the students to think of situations or activities which must be done in order for the desired outcome to be achieved.

Some possible suggestions are constructing a building, making a cake, or rebuilding an engine.

The same is true in mathematics.

Example 1: Solving a numerical expression with parentheses must begin within the grouping symbols.

$$\begin{array}{r} 2 \times (3 + 2) \\ 2 \times (5) \\ 10 \end{array}$$

Example 2: Solving a numerical expression where there are two or more grouping symbols must begin within the innermost grouping symbol.

$$\begin{array}{r} 3[4 + (2 - 1) \times 6] = \\ 3[4 + 1 \times 6] = \\ 3[4 + 6] = \\ 3[10] = \\ 30 \end{array}$$

ALGEBRA I

AREA: SIGNED NUMBERS

- OUTCOMES:
1. Given a number, identify its opposite value.
 2. Given a number, state its absolute value.

NOTES FOR THE TEACHER

Prerequisites for Students

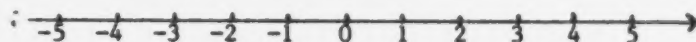
1. Know the meaning of absolute value, positive numbers, negative numbers, and real numbers.
2. Understanding the order of operations and simplifying parentheses.

Steps to Success

1. Every real number has one and only one opposite. The sum of a number and its opposite is zero.
2. Numbers such as (a) and $(-a)$ are called opposites or additive inverses.
3. The absolute value of a number is denoted by $|a|$.

ACTIVITY 1 - OUTCOMES 1 and 2

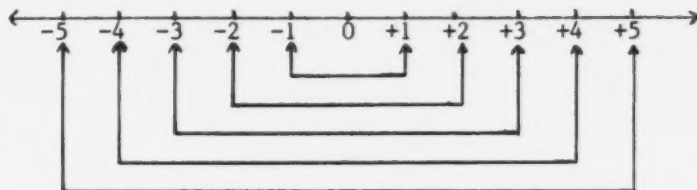
Draw this number line on the board.



On a number line, a number can be paired with another number which is the same distance from zero.

ACTIVITY 1 - OUTCOMES 1 and 2 (Cont'd)

Draw arrows from -1 to 1; -2 to 2, etc.



Notice that when each number has been paired with another number, they are on opposite sides of zero. These pairs of numbers are called "opposites."

-1 is the opposite of +1

-2 is the opposite of +2

+5 is the opposite of -5

NOTE: Zero is its own opposite

Write this algebraically by using the negative symbol - to mean "the opposite of."

$-(-1) = +1$ The opposite of -1 is +1

$-(-2) = +2$ The opposite of -2 is +2

$-(+5) = -5$ The opposite of +5 is -5

In every pair of opposite numbers other than 0, the positive number is always the greater. The absolute value of a non-zero number and its opposite is the same.

The absolute value of -6 is 6, algebraically written: $|-6| = 6$.

The absolute value of 6 is 6, algebraically written: $|6| = 6$.

NOTE: The absolute value of a positive number is always the number itself. The absolute value of a negative number is always its opposite.

ACTIVITY 1 - OUTCOMES 1 and 2 (Cont'd)

State whether the following sentences are true or false:

Example 1: $|150| = 150$ True

Example 2: $-(-.5) = -.5$ False

Example 3: $|+3.7| - |-3.7| = 0$ True

Example 4: $-[-(20 - 5)] = -15$ False

ALGEBRA 1

AREA: SIGNED NUMBERS

OUTCOME: 3. Solve problems involving addition, subtraction, multiplication, and division of signed numbers.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Be familiar with positive numbers, negative numbers, and absolute value.
2. Know that parentheses, are symbols of multiplication; and the bar, $\frac{\quad}{\quad}$, is a symbol of division when used in writing a number as a fraction, $\frac{x^2 + 2x + 1}{x + 1}$
3. Know that addition and subtraction are inverse operations, and that multiplication and division are also operations.

Steps for Success

1. The rules for addition, subtraction, multiplication, and division of signed numbers should be mastered.

ACTIVITY 1 - OUTCOME 3

TYPE 1: ADDITION OF SIGNED NUMBERS

Draw a tunnel on the board.



ACTIVITY 1 - OUTCOME 3 (Cont'd)

TYPE 1: ADDITION OF SIGNED NUMBERS (Cont'd)

Ask students to think of the tunnel entrance as a point represented by zero. The tunnel extends to the left indefinitely in equal units. The highway out of the tunnel to the right also extends indefinitely in equal units. Entering the tunnel means going in a negative direction. Leaving the tunnel means going in a positive direction.

Start at zero and travel 5 units away from the tunnel. From this point, travel 10 more units away from the tunnel.

Represent this movement algebraically.

$$+5 + 10 = 15 \quad \text{or} \quad \begin{array}{r} +5 \\ + +10 \\ \hline +15 \end{array}$$

Rule 1: The sum of two positive numbers is positive. Find the sum by adding their absolute values.

Example 1:
$$\begin{array}{r} +50 \\ + +50 \\ \hline +100 \end{array}$$

Example 2: $+8.1 + +3.2 = +11.3$

Example 3: $6 \frac{1}{2} + 3 \frac{1}{4} = 9 \frac{3}{4}$

Start at zero again, but this time travel into the tunnel 6 units. Travel 8 more units farther into the tunnel.

Represent this movement algebraically.

$$\begin{array}{r} -6 \\ + -8 \\ \hline -14 \end{array} \quad \text{or} \quad -6 + -8 = -14$$

ACTIVITY 1 - OUTCOME 3 (Cont'd)

RULE 2: The sum of two negative numbers is a negative number. The sum is found by adding the absolute values of the numbers.

Example 1:

$$\begin{array}{r} -30 \\ -40 \\ \hline -70 \end{array}$$

Example 2: $-8.35 + -2.10 = (-10.45)$

Example 3: $-13 \frac{1}{2} + -5 \frac{1}{4} = -18 \frac{3}{4}$

Next start at zero and travel 7 units away from the tunnel. Then travel 3 units toward the tunnel.

Ask: Can this be written algebraically?

$$\begin{array}{r} +7 \\ + -3 \\ \hline +4 \end{array} \quad \text{or} \quad +7 + (-3) = +4$$

Start at zero and go to the right 8 units. Travel to the left 10 units. Can this be written algebraically?

$$\begin{array}{r} +8 \\ + -10 \\ \hline -2 \end{array} \quad \text{or} \quad +8 + (-10) = -2$$

ACTIVITY 1 - OUTCOME 3 (Cont'd)

RULE 3: To find the sum of a positive number and a negative number, find the difference in the absolute values of the two numbers. The sum is positive if the positive number has the greater absolute value. The sum is negative if the negative number has the greater absolute value.

Example 1:

$$\begin{array}{r} +38 \\ + -2 \\ \hline +36 \end{array}$$

Example 2:

$$\begin{array}{r} +12 \\ + -24 \\ \hline -12 \end{array}$$

Example 3: $12.5 + (-.5) = 12$

Example 4: $-15 \frac{2}{3} + 1 \frac{2}{3} = -14$

ACTIVITY 2 - OUTCOME 3

TYPE 2: MULTIPLICATION OF SIGNED NUMBERS

Ask the students to imagine they are making a video tape of their neighborhood with a portable VCR camera. A neighbor jogs down the street, and they record the action. Soon another neighbor comes into view, but he is running backwards. They tape this action.

Imagine what is seen on the TV screen when viewing the first runner. (a neighbor running forward down the street)

$$\begin{array}{ccccccc} \text{VCR (Forward)} & \text{Times} & \text{Neighbor Running Forward} & & \text{Positive Forward Motion} \\ (+) & \times & (+) & = & + \end{array}$$

Translated algebraically: $(+a)(+b) = (+ab)$

Example 1: $(+2)(+40) = +80$

Example 2: $\left(+\frac{5}{8}\right)\left(+\frac{1}{2}\right) = +\frac{5}{16}$

ACTIVITY 2 - OUTCOME 3 (Cont'd)

TYPE 2: MULTIPLICATION OF SIGNED NUMBERS (Cont'd)

What is seen on the TV screen when viewing the second runner? (a neighbor running backwards down the street)

VCR (Forward)	Times	Neighbor Running Backwards	Negative Motion
(+)	x	(-)	= -

Translated algebraically: $(+a)(-b) = -ab$

Example 1: $(+.4)(-.3) = -.12$

Example 2: $(+\frac{3}{4})(-\frac{1}{8}) = -\frac{3}{32}$

What is seen on the TV screen when viewing the first runner with the VCR on reverse (rewind)? (The runner who was actually running forward appears to be running backwards.)

VCR (Reverse/Rewind)	Times	Neighbor Running Forward	Negative Motion
(-)	x	(+)	= -

Translated algebraically: $(-a)(+b) = -ab$

Example 1: $(-60)(+3) = -180$

Example 2: $(-\frac{1}{8})(+\frac{2}{3}) = -\frac{1}{12}$

TYPE 2: MULTIPLICATION OF SIGNED NUMBERS (Cont'd)

What is seen on the TV screen when viewing the second neighbor running backwards with the VCR playing on reverse (rewind)? (The runner who was actually running backwards appears to be running forward.)

VCR (Reverse/Rewind)	Times	Neighbor Running Backward	Positive Motion
(-)	x	(-)	= +

Translated algebraically: $(-a)(-b) = +ab$

ACTIVITY 2 - OUTCOME 3 (Cont'd)

Example 1: $(-4.5)(-2) = +9.0$

Example 2: $(-14)\left(-\frac{1}{2}\right) = +7$

RULES FOR MULTIPLICATION OF POSITIVE AND NEGATIVE NUMBERS

1. The product of two positive numbers or two negative numbers is a positive number.
2. The product of a positive number and a negative number is a negative number.

NOTE When there are more than two negative factors involved, the product of an even number of negative numbers is positive; and the product of an odd number of negative numbers is negative.

Example 1: $(-2)(-3)(-4)(-1) = +24$
 $(+6)(-4)(-1) =$
 $(-24)(-1) =$
 $+24 =$

Example 2: $(-8)(-2)(-3) = -48$
 $(+16)(-3) =$
 $-48 =$

ACTIVITY 2 - OUTCOME 3 (Cont'd)

TYPE 3: DIVISION OF SIGNED NUMBERS

Since division is the inverse operation of multiplication, the same rules apply for division as multiplication.

$$1. \frac{+a}{+b} = +\frac{a}{b}$$

$$\text{Example 1: } \frac{+20}{+10} = +2$$

$$2. \frac{+a}{-b} = -\frac{a}{b}$$

$$\text{Example 2: } \frac{+15}{-3} = -5$$

$$3. \frac{-a}{+b} = -\frac{a}{b}$$

$$\text{Example 3: } \frac{-2.5}{.5} = -5$$

$$4. \frac{-a}{-b} = +\frac{a}{b}$$

$$\text{Example 4: } \frac{-14}{-2} = +7$$

TYPE 4: SUBTRACTION OF SIGNED NUMBERS

Subtraction is the inverse operation of addition.

In arithmetic, subtraction involves only positive numbers, such as the following:

$$\begin{array}{r} \text{Example 1: } 6.5 \\ - 4.0 \\ \hline 2.5 \end{array}$$

$$\text{Example 2: } 826 - 126 = 700$$

In algebra, the negative numbers are included. Subtraction may appear in two different cases.

Case 1: Vertically written

$$\begin{array}{r} \text{Example 1: } (+8) \\ - (+2) \\ \hline \end{array}$$

$$\text{is the same as } \begin{array}{r} 8 \\ - 2 \\ \hline 6 \end{array}$$

ACTIVITY 2 - OUTCOME 3 (Cont'd)

Example 2:

$$\begin{array}{r} \text{Subtract } +9 \\ - +6 \\ \hline \end{array}$$

is the same as

$$\begin{array}{r} +9 \\ +(-6) \\ \hline +3 \end{array}$$

Case 2: Horizontally written

$$\begin{array}{l} \text{Example 1: } (+20) - (+18) = \\ \quad +20 - 18 = +2 \end{array}$$

$$\begin{array}{l} \text{Example 2: } (-11) - (-12) = \\ \quad -11 + 12 = 1 \end{array}$$

$$\begin{array}{l} \text{Example 3: } (-1.5) - (+.2) = \\ \quad -1.5 + -.2 = -1.7 \end{array}$$

RULE FOR SUBTRACTION:

The signed number is subtracted by adding its opposite to the minuend.

ALGEBRA I

AREA: SIGNED NUMBERS

- OUTCOMES: 4. Translate a word problem into mathematical expressions or equations using the correctly signed numbers.
5. Solve simple word problems involving signed numbers.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Have a knowledge of signed numbers.
2. Know the signs of equality and inequality.

Step to Success

1. The key to translating word problems into mathematical expressions is the student's ability to recognize pertinent facts and represent them with positive or negative numbers.

ACTIVITY 1 - OUTCOMES 4 and 5

Do you remember playing "May I?" when you were a youngster? The outcome of the game depended upon your ability to follow directions. For example, the command, TAKE THREE GIANT STEPS, was given. Then you had to say, "May I?". What happened if you didn't follow directions? You had to go back to the beginning.

We are going to play "May I?" with signed numbers. Don't forget you have to follow directions exactly.

Directions:

I will make a statement. You will write it down algebraically. As soon as you have completed the problem, hold up your hand. I will call your name. You should say, "May I?". Then I will say, "Yes, you may put it on the board."

I will read the problem twice.

ACTIVITY 1 - OUTCOMES 4 and 5 (Cont'd)

Example 1: An elevator starts at the ground floor and goes to the seventh floor. It then goes down three floors. Where is the elevator now?

Repeat Example 1.

Call on the first student who raises his hand. If that student responds with "May I?", say, "Yes, you may put the problem on the board."

$$+7 + (-3) = 4$$

It is on the fourth floor.

Example 2: John drove to a point 85 miles east of his home. He then drove 120 miles west. At what point is he now with respect to his home?

Repeat Example 2.

Call on the first student who raises his hand. If the student replies, "May I?", ask the student to put the problem on the board.

$$+85 + (-120) = -35$$

He is 35 miles west of his home.

Example 3: A golfer played four rounds of golf. The golfer scored 1 under par, 3 over par, 6 over par, and 2 under par. How did the golfer's four rounds finish with respect to par?

Follow same procedure for getting participation.

$$-1 + 3 + 6 + (-2) = 6$$

The golfer was 6 over par.

ACTIVITY 1 - OUTCOMES 4 and 5 (Cont'd)

Example 4: A running back carried the ball six times in a ballgame. On succeeding carries he gained 7 yards, gained 22 yards, lost 2 yards, gained 28 yards, gained 45 yards, and lost 5 yards. What was the net gain on the 6 carries?

Follow the same procedure:

$$+7 + 22 - 2 + 28 + 45 - 5 = 95$$

He gained 95 yards.

NOTE: Assign each problem a value of one point and set a goal to declare a winner. Continue the game until a sufficient number of examples have been solved for a student to achieve the set goal and to be declared the winner of "May I?" with signed numbers.

ALGEBRA I

AREA: OPEN PHRASES, OPEN SENTENCES, AND SOLUTION SETS

- OUTCOMES:
1. Use mathematical terminology to translate an equation in various equivalent English sentences.
 2. Define and recognize the terms: base, exponent, and power.
 3. Determine the degree of monomials and polynomials.
 4. Determine the solution set of a given open sentence.
 5. Translate words into variable expressions and simplify the expression when necessary.
 6. Translate words into equations and simplify the equation when necessary.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Know the meanings of real numbers and variables.
2. Know the symbols for equality and inequality.
3. Extend knowledge of the number line to all real numbers.
4. Use the procedure for translating words into variable expressions or equations that was used in expressing number relationships with mathematical symbols.
5. Use the understanding of words, such as "more than," "product," "decreased by," "diminished by," "exceeds," "divided by," "sum of", and their relationships to the symbols of operation to enhance the student's ability to translate words into algebraic expressions.
6. Understand that symbols of division and multiplication may be written several ways.

Example 1: 100 divided by A

- a. $100 \div A$
- b. $100/A$

Example 2: 5 times B

- a. $5 \times B$
- b. $5 \cdot B$
- c. $5(B)$
- d. $5B$

Steps to Success

1. An understanding of the following is essential:

An open sentence is a statement containing an unknown.

Replacement set is the set whose members may be used to replace the variable or unknown.

Solution set is the set that consists of all members of the replacement set that make the sentence true.

Null set or Empty set is the set with no members.

2. Inequalities are graphed as follows:



greater than $>$

less than $<$

greater than or equal to $\geq$

less than or equal to $\leq$

ACTIVITY 1 - OUTCOMES 1 and 2

To represent accurately an expression algebraically, extend vocabulary to include words that may not have been used before.

Vocabulary:

1. Monomial: A numeral or variable or both which are connected by multiplication or division

Examples:

a. 150

b. a

c. $a \div 150$ or $\frac{a}{150}$

d. 150A or A(150)

ACTIVITY 1 - OUTCOMES 1 and 2 (Cont'd)

2. Polynomial: An algebraic expression containing more than one term separated by + and/or - signs

Examples: a. $a + 2$

b. $\frac{a}{b} + 2$

c. $\left(\frac{a-b}{3}\right) + 2$

d. $12a + 3b - 14c$

3. Factor: Each of the numbers that are multiplied to get a product

Example: $6ab$

Think of different combinations of numbers that can be multiplied to get $6ab$.

a. $(1)(6ab)$

f. $(6a)(b)$

b. $(2)(3ab)$

g. $(3)(2ab)$

c. $(6)(ab)$

h. $(3)(2a)(b)$

d. $(a)(6b)$

i. $(3a)(2b)$

e. $(2a)(3b)$

j. $(3)(2)(a)(b)$

All of these are factors of $6ab$.

4. Coefficient: Any factor is the coefficient of the other factor(s) in a product

Example 1: $6ab$

a. 6 is the coefficient of ab

b. $6a$ is the coefficient of b

c. $6b$ is the coefficient of a

d. 6 is the numerical coefficient of ab

e. ab is the literal coefficient of 6

Example 2: x

The value of x is the same as $1 \cdot x$. When a variable does not have an expressed numerical coefficient, the coefficient of x is understood to be 1.

ACTIVITY 1 - OUTCOMES 1 and 2 (Cont'd)

5. Base: A number which is used as a factor

Exponent: A number which tells how many times the base is used as a factor

Power: The product derived

It is advantageous to use short cuts in algebra when applicable. For example, why write the product of 3 ten times, $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = \underline{\hspace{1cm}}$, when a shorter version could be used? The product of 3 ten times can be written using exponents.

Base $\rightarrow 3^{10}$ $\leftarrow$ exponent = 59,049 $\leftarrow$ power

Example 1: a. $5^1 = 5$
b. $5^2 = 5 \times 5 = 25$
c. $5^3 = 5 \times 5 \times 5 = 125$

Example 2: a. $a^1 = a$ (a to 1st power)
b. $a^2 = a \times a$ (a squared)
c. $a^3 = a \times a \times a$ (a cubed)
d. $a^4 = a \times a \times a \times a$ (a to the 4th power)

ACTIVITY 2 - OUTCOMES 1 and 6

Ask students what the following statement means: "She made an A on the last exam."

As students attempt to clarify the statement, the following questions will arise:

Whom are we talking about?

What was her grade on the last exam?

This statement is called an open sentence. More information must be known before it can be determined if it is a true or false statement.

What must be known? The name of the person being talked about. After replacing the word "she" with a name, it can be determined if this statement is true or false.

ACTIVITY 2 - OUTCOMES 1 and 6 (Cont'd)

For example, all female students in this class may be listed as a possibility to make the statement a true statement. The names of the female students are the replacement set. Now there are several possibilities for answers:

- a. Only one girl made an A.
- b. Several girls made an A.
- c. All girls made an A.
- d. No one made an A.

The answer is called the solution set.

In algebra the symbol used to represent an unknown is called a variable. The equation, $x + 2 = 5$, has x as the unknown variable until it is replaced with a number. The set from which the answer is chosen is called the replacement set or domain of the variable. When the replacement set for x is $\{1, 2, 3, 4, 5\}$, the solution can come from only those numbers which are in the set.

Is the number that will make $x + 2 = 5$ a true statement in the replacement set? Yes, $3 + 2 = 5$; therefore, $x = \{3\}$ is the solution set.

Is the number that will make $x - 5 = 10$ a true statement in this replacement set? There is not a number in the replacement set that would make $x - 5 = 10$ true; therefore, the solution set is the empty set or null set. The symbol of an empty set is $\emptyset$ or $\{ \}$.

Example 1: Which of these statements are open sentences?

- a. He is a tennis player.
- b. _____ is president of the student body.
- c. $5 + 2 = 4 + 3$
- d. $x + 8 = 55$

Example 2: What is the variable in each sentence?

- a. The population of our school is n students.
- b. She won the academic award.
- c. $R - .5 = 12$
- d. $14C = 56$

ACTIVITY 2 - OUTCOMES 1 and 6 (Cont'd)

Example 3: Given the replacement set $\{1, 1.5, 2, 2.5, 3, 3.5\}$, find the solution set.

a. $4D = 4$

b. $2Y = 5.0$

c. $W - 3.5 = 0$

d. $10 - P = 7$

e. $B - 5.5 = 7$

f. $O(A) = 0$

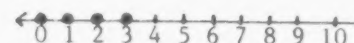
ACTIVITY 3 - OUTCOMES 1 and 4

Look at solution sets as they would appear on a graph. Graph the solution set when the replacement set is: $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

Example 1: $A + 3 = 6$

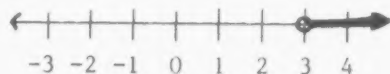


Example 2: $A - 2 < 6$

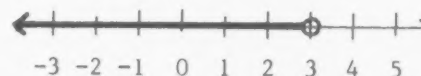


Don't stop with just whole numbers. Look at graphing solution sets when the replacement set is the {real numbers}.

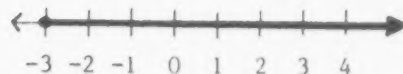
Example 3: $B > 3$



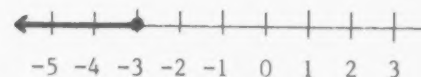
Example 4: $B < 3$



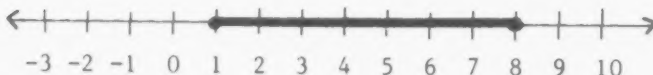
Example 5: $B \geq -3$



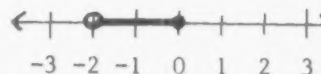
Example 6: $B \leq -3$



Example 7: $1 \leq c \leq 8$



Example 8: $-2 < c \leq 0$



ACTIVITY 4 - OUTCOMES 5 and 6

Ask students what the following drawing means.



- J. B. + S. R.
- J. B. likes S. R.
- J. B. loves S. R.

Whatever the case, it is understood to mean that at some point J. B. or S. R. cared for the other, or maybe even both cared for each other. These are symbols for a verbal phrase. Mathematical symbols can also be used to translate words into variable expressions such as:

Examples:

1. 8 less than y
2. 6 more than x
3. 4 times w
4. 150 diminished by 8 times B
5. The sum of c and d divided by e
6. s squared

Solutions:

$$\begin{array}{l} y - 8 \\ 6 + x \\ 4w \\ 150 - 8B \\ \frac{c + d}{e} \\ s^2 \end{array}$$

Now, use mathematical symbols to solve problems in which letters represent variables.

Examples:

1. A computer costs \$999. Represent the cost of M computers. 999 M
2. A man bought a car and a motorcycle for a total of \$15,840. If the motorcycle cost N dollars, represent the cost of the car. \$15,840 - N

ACTIVITY 4 - OUTCOMES 5 and 6 (Cont'd)

3. The width of a rectangle is W meters. Represent the length which is 25 meters longer. $W + 25$
4. Represent the number of cents in d dimes. $.10d$
5. The volume of a cube is e cubed. Represent the volume if e is 1.57 cm. $(1.57)^3$

ACTIVITY 5 - OUTCOME 3

For everyone all over the world to write expressions so that other people will determine the same meaning, expressions must be written in order.

Before an expression can be written in order, the degree of its terms must be known. The degree of a monomial is the sum of the degree of its variables. Find the degree of each monomial:

Example 1: x^3 degree of 3

Example 2: $3x^4$ degree of 4

Example 3: xy^4 degree of 5

Example 4: $2x^2y^3$ degree of 5

The degree of a polynomial is the degree of the term of highest degree.

$$\begin{array}{ccccc} 3x^3 & + & 4xy & - & 3x^2y^3 \\ \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} \\ \text{degree} & & \text{degree} & & \text{degree} \\ \text{of 3} & & \text{of 2} & & \text{of 5} \end{array}$$

Therefore, the degree of the polynomial is 5.

ACTIVITY 5 - OUTCOME 3 (Cont'd)

Find the degree of each polynomial:

Example 1: $2y^2 + 3$ degree of 2

Example 2: $8x^3 + 3y^2$ degree of 3

Example 3: $3t^2 + 2t + 1$ degree of 2

It is easier to work with polynomials when they are arranged in order. In most cases, arrange the terms in decreasing degree of one variable.

Example: $a^2 + a^3 + 1 + a$ would be $a^3 + a^2 + a + 1$

Notice the powers of a are in decreasing order.

Sometimes an expression has more than one variable. In this case, place it in order of degree in one variable.

Example: $8ab + 4a^2 + 2b^2$ would be $4a^2 + 8ab + 2b^2$

Notice the powers of a are in decreasing order while the powers of b are in increasing order.

ALGEBRA I

AREA: POSTULATES AND PROPERTIES OF OPERATIONS

- OUTCOMES:
1. Define the reflexive, symmetric, and transitive properties for all real numbers a , b , and c .
 2. Write mathematical expressions demonstrating the reflexive, symmetric, and transitive properties.
 3. Define closure for all real numbers a and b .
 4. Write a mathematical equation demonstrating the commutative property.
 5. Identify the commutative property given a mathematical equation expressing the property.
 6. Write a mathematical equation demonstrating the associative property.
 7. Identify the associative property given a mathematical equation expressing the property.
 8. Given an expression, be able to identify another way of writing the expression.
 9. Define the identity property for real numbers.
 10. Write a mathematical expression demonstrating the identity property of addition and multiplication.
 11. Recognize one as the multiplicative identity.
 12. Define the distributive property.
 13. Name the missing addend in a number sentence illustrating the distribution of multiplication over addition.
 14. Recognize an application of the distribution of multiplication over addition.
 15. Define the multiplicative inverse.
 16. Recognize the reciprocal of a number.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Be familiar with natural numbers, integers, even numbers, odd numbers, rational numbers, and whole numbers.
2. Know the order of operations.

Steps to Success

1. Students should understand that these mathematical properties are laws which tell us what can or cannot be done with numbers and/or expressions.
2. Students should understand that this initial list of properties is a back-up system to justify all algebraic manipulations.
3. Students should spell correctly and identify names of postulates and properties which are demonstrated algebraically.

ACTIVITY 1 - OUTCOMES 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15 and 16

TYPE 1: PROPERTIES OF ADDITION

1. Commutative Property of Addition: When adding numbers, the order can be reversed. Ask students what the word "commute" means. Attention should be drawn to the "back and forth" idea.

Commutative Property of Addition represented algebraically is: $a + b = b + a$.

Example 1: $6 + 8 = 8 + 6$

Example 2:
$$\begin{array}{r} 7 \\ + 2 \\ \hline 9 \end{array} \quad \text{or} \quad \begin{array}{r} 2 \\ + 7 \\ \hline 9 \end{array}$$

2. Associative Property of Addition: When adding numbers, the addends can be regrouped.

What is implied by the phrases: "Doctors Smith, Jones, and Associates," or "National Rifle Association," or "Alabama Education Association," or "I will not associate with you anymore!"

Attention should be drawn to the words "group" or "together."

The Associative Property of Addition represented algebraically is $(a + b) + c = a + (b + c)$.

Notice the groups can change.

Example : $(-3 + 2) + 5 = -3 + (2 + 5)$
By order of operations $(-1) + 5 = -3 + 7$
 $4 = 4$

3. The Identity Property of Addition: When adding zero, the sum is identical to the sum of the other addend(s).

Ask students what is implied by words like identification, identify, and identical twins.

ACTIVITY 1 - OUTCOMES 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15 and 16 (Cont'd)

Attention should be drawn to the idea of "the same as."

Identity Property of Addition represented algebraically is $a + 0 = a$.

Example: $3 + 0 = 3$

NOTE: The Identity Property guarantees that if 0 is added to 3, the answer will be 3; therefore, the identity element for addition is 0.

4. Inverse Property for Addition: When a number is added to its inverse, the result is the identity element.

Ask students what the word "inverse" means.

Attention should be drawn to the idea of "undo" or "opposite."

The Inverse Property for Addition stated algebraically is $a + (-a) = 0$.

Example: $2 + (-2) = 0$

Note: Any number added to its opposite is zero.

TYPE 2: PROPERTIES OF MULTIPLICATION

1. Commutative Property of Multiplication

Ask students what the word, "commute" means and refer them to the Commutative Property of Addition. Then ask students to state the Commutative Property for Multiplication, i.e., $a \cdot b = b \cdot a$.

Example: $4 \cdot 5 = 5 \cdot 4$
 $20 = 20$

Numerous examples of this property are in the multiplication tables.

ACTIVITY 1 - OUTCOMES 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15 and 16 (Cont'd)

2. Associative Property of Multiplication

Ask students what the words "associate" and "associates" mean and refer them to the Associative Property of Addition. Then ask students to state the Associative Property of Multiplication, i.e., when multiplying, the grouping may change.

Algebraically written it is: $(ab)c = a(bc)$.

Example: $(-2 \cdot 5)4 = -2(5 \cdot 4)$
 $(-10)4 = -2(20)$
 $-40 = -40$

3. Identity Property of Multiplication

Ask students what number they can multiply any other number by to get the same number.

Algebraically written it is: $a \cdot 1 = a$.

Example: $\frac{1}{5} \cdot 1 = \frac{1}{5}$

Therefore, the identity element for multiplication is 1.

4. Inverse Property of Multiplication

Ask students what they can multiply a number by to get the identity element.

A number is multiplied by its reciprocal to get the identity element.

Algebraically written it is $a \cdot \frac{1}{a} = 1$.

ACTIVITY 1 - OUTCOMES 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15 and 16 (Cont'd)

Note that multiplicative inverses are reciprocals.

Example: $-2 \left(-\frac{1}{2} \right) = 1$

Therefore, the reciprocal of -2 is $-\frac{1}{2}$.

Emphasize that any number multiplied by its reciprocal gives a product of 1.

The reciprocal of $\frac{1}{5}$ is 5 because $\frac{1}{5} \cdot 5 = 1$.

TYPE 3: THE PROPERTY OF BOTH ADDITION AND MULTIPLICATION SIMULTANEOUSLY

Distributive Property of Multiplication with Respect to Addition is a law which states: When multiplying a number by a sum, the number must be multiplied by every addend.

Ask students what the words "distribute," "distribution," or "distributive" mean.

Attention should be drawn to the idea of "handing out."

Algebraically written it is $a(b + c) = ab + ac$.

Example 1: $4(-2 + 1) = 4(-2) + 4(1)$
 $= -8 + 4$
 $= -4$

Example 2: $2(x + 1) = 2(x) + 2(1)$
 $= 2x + 2$

ACTIVITY 2 - OUTCOMES 1, 2 and 3

1. Reflexive Property of Equality: Any number equals itself.

Ask students what looks back if you look in a mirror. (the reflection or the same)

Algebraically written it is $a = a$.

Example: $125 = 125$

2. Symmetric Property of Equality: If two numbers are equal, you may reverse the order.

Butterfly wings and leaves are examples of some things that are identical if folded down the middle. This is symmetry in nature. In mathematics, a circle is a good example of symmetry.

Algebraically written, if $a = b$, then $b = a$.

Example: $a = 3$, $3 = a$ (fold in middle)

3. Transitive Property of Equality: If one number equals a second number, and the second number equals a third number; then the first number must equal the third number.

Ask students what is implied by the words "transfer," "transit," "transport," and "transportation."

Attention should be drawn to the idea of "moving."

Algebraically written: if $a = b$, and $b = c$; then $a = c$.

Example 1: If Tom's age equals 14, and 14 equals Sally's age; then Tom's age equals Sally's age.

Example 2: If $a = 2$, and $2 = b$; then $a = b$.

ACTIVITY 2 - OUTCOMES 1, 2 and 3 (Cont'd)

TYPE 2: CLOSURE PROPERTY

1. The Closure Property: In a given set, if an operation is performed on elements in the set, the result must also be in the set.

Ask students the meaning of the word "closed."

Attention should be drawn to the idea of "no escape."

Ask students to pick two even numbers and add them, and repeat several times. What is always the result?

(2) Choose two odd numbers and add them. Repeat several times. What is always the result?

Therefore, even numbers are closed or have the Closure Property for Addition. Odd numbers do not.

Algebraically written: the Closure Property of Real Numbers ($\mathbb{R}$) is: If $a \in \mathbb{R}$ and $b \in \mathbb{R}$ then $a + b \in \mathbb{R}$ and $a \times b \in \mathbb{R}$.

ALGEBRA I

AREA: OPERATIONS WITH MONOMIALS AND POLYNOMIALS

- OUTCOMES:
1. Add and subtract single term algebraic expressions.
 2. Add and subtract multiple term algebraic expressions.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Have an understanding of coefficients, exponents, like terms, monomials, polynomials, degree of a monomial, degree of a polynomial, decreasing and increasing order.
2. Know the rules for adding and subtracting signed numbers.
3. Know the commutative and associative properties for addition and multiplication and the distributive property for multiplication over addition.

Steps to Success

1. The student must be able to recognize similar (like) terms.
2. The students will learn to follow rules for adding or subtracting similar terms by reciting the rules as they use them.
3. Students need to be aware of the fact that they are writing the variables down and adding or subtracting the coefficients.

ACTIVITY 1 - OUTCOME 1

Look at the graph and answer the following questions.

x = 10 students

Class	Number of Students
Algebra	xxx
P. E.	xxxxx
French	xx

ACTIVITY 1 - OUTCOME 1 (Cont'd)

- (a) How many students are in the Algebra and the French class?

Solution

$$\begin{array}{r} \text{Algebra} \quad 3x \\ \text{French} \quad + 2x \\ \hline 5x \end{array}$$

5(10) = 50 students in both classes

- (b) How many more students are in the P. E. class than are in the Algebra class?

Solution

$$\begin{array}{r} \text{P. E.} \quad 5x \\ \text{Algebra} \quad - 3x \\ \hline 2x \end{array}$$

2(10) = 20 students more in P. E. than in Algebra.

ACTIVITY 2 - OUTCOME 2

Combine the following terms: $3a + 4ab - 6ab + 7a - ab + 3ab$

Use colored chalk to underline like terms. Have students tell which color to underline each term. Put like terms in columns and compute the answer.

$$\begin{array}{rcl} 3a & + 4ab & - 6ab \\ + 7a & - ab & + 3ab \\ \hline 10a & + 3ab & - 3ab \end{array}$$

ALGEBRA I

AREA: OPERATIONS WITH MONOMIALS AND POLYNOMIALS

- OUTCOMES:
3. Multiply single term algebraic expressions.
 4. Multiply a multiple term algebraic expression by a single term algebraic expression.
 5. Multiply a multiple term algebraic expression by a multiple term algebraic expression.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Have an understanding of coefficients, exponents, like terms, monomials, polynomials, trinomials, degree of a monomial, degree of a polynomial, decreasing and increasing order.
2. Know the commutative and associative properties for addition and multiplication and the distributive property for multiplication over addition.

Steps to Success

1. To multiply monomials, use the rules of exponents with the commutative and associative properties.
2. To multiply monomials with the same base, add their exponents. Products having factors of different bases are in simplest form.
3. To multiply a monomial times a polynomial, use the distributive property with the rules of exponents.
4. To multiply polynomials, use the distributive property and multiply each term in one polynomial times each term in the other polynomial.

ACTIVITY 1 - OUTCOME 3

When multiplying monomials, three cases may occur.

TYPE 1: MONOMIALS WITH THE SAME BASE

$$b^m \cdot b^n = b^{m+n}$$

$$\text{Example 1: } x^2 \cdot x^3 = x^{2+3} = x^5$$

$$\text{Example 2: } 4a \cdot 2a^3 = (4 \cdot 2)(a \cdot a^3) = 8a^{1+3} = 8a^4$$

$$\text{Example 3: } b^y \cdot b^2 = b^{y+2}$$

$$\text{Example 4: } c^2d^3 \cdot cd^4 = (c^{2+1})(d^{3+4}) = c^3d^7$$

$$\begin{aligned} \text{Example 5: } 8a^2b^3c \cdot 2b^2c^2 &= \\ (8 \cdot 2)(a^2)(b^3 \cdot b^2)(c^1 \cdot c^2) &= \\ 16a^2(b^3+2)(c^1+2) &= \\ 16a^2b^5c^3 \end{aligned}$$

TYPE 2: POWER OF A PRODUCT

$$(ab)^m = a^mb^m$$

$$\text{Example 1: } (cd)^2 = c^2d^2$$

$$\text{Example 2: } (-2f)^3 = (-2)^3(f)^3 = -8f^3$$

TYPE 3: POWER OF A POWER

$$(b^m)^n = b^{mn}$$

$$\text{Example 1: } (r^2)^3 = r^{2 \cdot 3} = r^6$$

$$\text{Example 2: } (a^2b^{10})^3 = (a^2)^3(b^{10})^3 = a^{2 \cdot 3}b^{10 \cdot 3} = a^6b^{30}$$

ACTIVITY 1 - OUTCOME 3 (Cont'd)

NOTE: Exponents are multiplied only when there is a power of a power.

It is important for the student to recognize the difference in the power of a product as in Type 2 and the power of a power as in Type 3.

ACTIVITY 2 - OUTCOME 4

To multiply a polynomial by a monomial, multiply each term in the polynomial by the monomial.

Example 1: $a(b + c) = ab + ac$

Example 2: $3a(a + 2) = 3a(a) + 3a(2)$
 $= 3a^2 + 6a$

Example 3: $2x^2(x + y + 2) = 2x^2(x) + 2x^2(y) + 2x^2(2)$
 $= 2x^3 + 2x^2y + 4x^2$

ACTIVITY 3 - OUTCOME 5

To multiply one polynomial by another, use the distributive property, multiply each term of one polynomial by each term of the other, and combine like terms.

Example 1: $(a + b)(a + b)$
 $a(a + b) + b(a + b)$
 $a^2 + ab + ab + b^2$
 $a^2 + 2ab + b^2$

Example 2: $(x + 3)(x + 2)$
 $x(x + 2) + 3(x + 2)$
 $x^2 + 2x + 3x + 6$
 $x^2 + 5x + 6$

ACTIVITY 3 - OUTCOME 5 (Cont'd)

Example 3:

$$\begin{aligned} & (x + 1)(x^2 + 2 - 1) = \\ & x(x^2 + 2x - 1) + 1(x^2 + 2x - 1) = \\ & \quad x^3 + 2x^2 - x + x + 2x - 1 = \\ & \quad \quad x^3 + 3x + x - 1 = \end{aligned}$$

Some students may find it easier to use the following method:

Example 4: $(a + b + c)(2a + 3b - c) =$

$$\begin{array}{r} 2a + 3b - c \\ a + b + c \\ \hline 2a^2 + 3ab - ac \\ \quad + 2ab \qquad + 3b^2 - bc \\ \qquad + 2ac \qquad + 3bc - c^2 \\ \hline 2a^2 + 5ab + ac + 3b^2 + 2bc - c^2 \end{array}$$

ALGEBRA I

AREA: OPERATIONS WITH MONOMIALS AND POLYNOMIALS

- OUTCOMES: 6. Divide single term algebraic expressions.
7. Divide a multiple term algebraic expression by a single term algebraic expression.
8. Divide a multiple term algebraic expression by a multiple term algebraic expression.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Know the substitution, commutative, and associative properties.
2. Be able to recognize coefficients, exponents, like terms, monomials, and polynomials.

Steps to Success

1. Know that the rules for division of exponents are

$$\frac{b^m}{b^n} = b^{m-n} \quad \text{if } m > n$$

and

$$\frac{b}{b^n} = \frac{1}{b^{n-1}} \quad \text{if } n > 1$$

2. Be able to develop the concept of $b^0 = 1$ and $b^n = \frac{1}{b^{-n}}$ or $b^{-n} = \frac{1}{b^n}$.
3. Know that in dividing a polynomial by a monomial, each term of the polynomial is divided by monomial, and the quotients are added.
4. Recognize that division of polynomials uses a similar procedure as that used in dividing whole numbers. The divisor and dividend must be in the same order.

ACTIVITY 1 - OUTCOME 6

There are two simple rules for dividing monomials.

Rule 1: $\frac{b^m}{b^n} = b^{m-n}$ if $m > n$

Example: $\frac{b^5}{b^2} = \frac{\cancel{b} \cancel{b} b b b}{\cancel{b} \cancel{b}} = b^3$

because $\frac{b^5}{b^2} = b^{5-2} = b^3$

Rule 2: $\frac{b^m}{b^n} = \frac{1}{b^{n-m}}$ if $n > m$

Example: $\frac{b^2}{b^5} = \frac{\cancel{b} \cancel{b}}{\cancel{b} \cancel{b} b b b} = \frac{1}{b^3}$

because $\frac{b^2}{b^5} = \frac{1}{b^{5-2}} = \frac{1}{b^3}$

Another helpful property is the quotient property.

$$\frac{ab}{cd} = \frac{a}{c} \cdot \frac{b}{d}$$

ACTIVITY 1 - OUTCOME 6 (Cont'd)

This will help simplify the division of monomials consisting of more than one like factor.

Example 1: $\frac{16 w^4 v^2}{-4 w^3 v} =$

$$\left(\frac{16}{-4}\right)\left(\frac{w^4}{w^3}\right)\left(\frac{v^2}{v}\right) =$$

$$-4 (w^{4-3})(v^{2-1}) =$$

$$-4 w^1 v^1 =$$

$$-4wv$$

Example 2: $\frac{-20 c^{10} d^2 e}{-10 c^5 d^8} =$

$$\left(\frac{-20}{-10}\right)\left(\frac{c^{10}}{c^5}\right)\left(\frac{d^2}{d^8}\right)\left(\frac{e}{1}\right) =$$

$$\left(\frac{+2}{1}\right)\left(\frac{c^{10-5}}{1}\right)\left(\frac{1}{d^{8-2}}\right)\left(\frac{e}{1}\right) =$$

$$\left(\frac{+2}{1}\right)\left(\frac{c^5}{1}\right)\left(\frac{1}{d^6}\right)\left(\frac{e}{1}\right) =$$

$$\frac{2 c^5 e}{d^6}$$

ACTIVITY 1 - OUTCOME 6 (Cont'd)

Two special cases will appear.

Case 1: Zero exponents

What happens in the fraction $\frac{b^m}{b^n}$ if m and n are equal?

$$\text{For example: } \frac{b^5}{b^5} = \frac{\cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b}}{\cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b}} = 1$$

$$\text{then } \frac{b^5}{b^5} = b^{5-5} = b^0 = 1$$

Case 2: Negative exponents

Find the value of $\frac{x^a}{x^b}$.

What is the solution?

It may be x^{a-b} or $\frac{1}{x^{b-a}}$.

If $a = 16$ and $b = 12$, use the form $x^{a-b} = x^{16-12} = x^4$.

But what if we use $\frac{1}{x^{b-a}} = \frac{1}{x^{12-16}} = \frac{1}{x^{-4}}$? Are x^4 and $\frac{1}{x^{-4}}$ equivalent expressions?

Substitute 10 for the value of x in both expressions.

$$x^4 = (10)$$

$$= 10,000$$

$$\begin{aligned} \frac{1}{x^{-4}} &= \frac{1}{10^{-4}} \\ &= \frac{1}{.0001} \\ &= 10,000 \end{aligned}$$

They are, therefore, equivalent expressions $b^m = \frac{1}{b^{-m}}$ or $b^{-m} = \frac{1}{b^m}$, and b^m and b^{-m} are reciprocals.

ACTIVITY 1 - OUTCOME 6 (Cont'd)

Express in simplest form with positive exponents.

$$\text{Example 1: } ab^{-3} = \frac{a}{b^3}$$

$$\text{Example 2: } \frac{e^0 d^{-1}}{e^{-2}} = \frac{1(e^2)}{d^1} = \frac{e^2}{d}$$

$$\begin{aligned} \text{Example 3: } 4p^2 + \frac{1}{p^{-2}} &= \\ 4p^2 + \frac{p^2}{5p^2} &= \end{aligned}$$

ACTIVITY 2 - OUTCOME 7

In algebra a monomial is not always divided by a monomial. Sometimes a polynomial is divided by a monomial. In most cases it will look exactly like a fraction in which the numerator contains two or more terms and the denominator is only one term.

$$\text{Example 1: } \frac{14x^5 + 8x^3 + 2x^2}{2x^2}$$

To simplify first divide each term in the numerator by the single term in the denominator.

$$\frac{14x^5}{2x^2} + \frac{8x^3}{2x^2} + \frac{2x^2}{2x^2}$$

Then use the rules for dividing monomials and simplify.

$$7x^3 + 4x + 1$$

ACTIVITY 2 - OUTCOME 7 (Cont'd)

Example 2: $\frac{18 - 3r}{3r} =$

$$\frac{18}{3r} - \frac{3r}{3r} =$$

$$\frac{6}{r} - 1$$

Example 3: $\frac{25x^2 - 15x}{5x} + \frac{14x^2 - 21x}{7x} =$

$$\frac{25x^2}{5x} - \frac{15x}{5x} + \frac{14x^2}{7x} - \frac{21x}{7x} =$$

$$5x - 3 + 2x - 3 =$$

$$7x - 6$$

ACTIVITY 3 - OUTCOME 8

A third type of division problem in algebra is a polynomial divided by a polynomial.

Example 1:

$$864 \div 16$$

Whole Numbers

Example 2:

$$x^2 + 5x + 6 \div x + 3$$

Polynomials

Whole Numbers	Procedure	Polynomials
$16 \overline{)864}$	1. Write problem in long division form.	$x + 3 \overline{)x^2 + 5x + 6}$
$16 \overline{)864} \begin{array}{r} 5 \\ \hline \end{array}$	2. Divide the left number of the dividend by the left number of the divisor.	$x + 3 \overline{)x^2 + 5x + 6} \begin{array}{r} x \\ \hline \end{array}$
$16 \overline{)864} \begin{array}{r} 5 \\ (-)80 \\ \hline 6 \end{array}$	3. Multiply all the divisors by the partial quotient and subtract.	$x + 3 \overline{)x^2 + 5x + 6} \begin{array}{r} x \\ (-)x^2 + 3x \\ \hline 2x \end{array}$
$16 \overline{)864} \begin{array}{r} 5 \\ (-)80 \\ \hline 64 \end{array}$	4. Bring down the next number in the dividend.	$x + 3 \overline{)x^2 + 5x + 6} \begin{array}{r} x \\ (-)x^2 + 3x \\ \hline 2x + 6 \end{array}$
$16 \overline{)864} \begin{array}{r} 54 \\ (-)80 \\ \hline 64 \end{array}$	5. Divide the left number of the divisor into the left number of the new dividend.	$x + 3 \overline{)x^2 + 5x + 6} \begin{array}{r} x + 2 \\ (-)x^2 + 3x \\ \hline 2x + 6 \end{array}$
$16 \overline{)864} \begin{array}{r} 54 \\ (-)80 \\ \hline 64 \\ (-)64 \\ \hline \end{array}$	6. Multiply the second number of the quotient times the divisor and subtract.	$x + 3 \overline{)x^2 + 5x + 6} \begin{array}{r} x + 2 \\ (-)x^2 + 3x \\ \hline 2x + 6 \\ (-)2x + 6 \\ \hline \end{array}$

ACTIVITY 3 - OUTCOME 8 (Cont'd)

To check, simply multiply the quotient times the divisor and derive the dividend.

Check for Example 1:

$$\begin{array}{r} 54 \\ \times 16 \\ \hline 324 \\ 54 \\ \hline 864 \end{array}$$

Check for Example 2:

$$\begin{array}{r} x + 2 \\ \times x + 3 \\ \hline x^2 + 2x \\ + 3x + 6 \\ \hline x^2 + 5x + 6 \end{array}$$

Be careful to note that the divisor and dividend are both in the same order, i.e., arranged in decreasing or increasing order in the variable.

Example: $\frac{6x + 3x^2 + 1}{x + 2}$ would be $x + 2 \overline{)3x^2 + 6x + 1}$.

Be aware that in dividing polynomials, sometimes a term is missing in the dividend.

Example: $\frac{a^2 - 4}{a + 2}$ $a + 2 \overline{)a^2 - 4}$

Notice there is not an a^1 term in the dividend; therefore, a place must be held for that missing term, e.g.,

$a + 2 \overline{)a^2 + 0a - 4}$ and the division performed as in Example 2.

ALGEBRA I

AREA: SIMPLE EQUATIONS, INEQUALITIES, AND PROBLEMS

- OUTCOMES:
1. Solve an equation involving addition.
 2. Solve an equation involving subtraction.
 3. Solve an equation involving multiplication.
 4. Solve an equation involving division.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Be able to identify the opposite value of a given number.
2. Be able to identify the multiplicative inverse or reciprocal of a number.

Steps to Success

1. The key to solving equations is to think of the equation as a balance scale. When the weights are equal, both sides of the scale are level. When different weights are placed on one side or the other, the scales are uneven or unequal. The scales remain equal as long as weights of equal measures are placed on or taken away from both sides of the scale.
2. An equation is like the balance scale. Its equality does not change as long as the same number and operation are applied to both sides of the equation.
3. Since addition and subtraction are inverse operations, addition can be thought of as "undoing" an equation involving subtraction and subtraction can be used to "undo" an equation involving addition. To "undo" addition, subtract the number in the equation from both sides of the equation.

Operation: Addition $X + 2 = 4$

"Undo" by: Subtraction $X + 2 - 2 = 4 - 2$

Solution: $X = 2$

To "undo" subtraction, add the number in the equation to both sides of the equation.

Operation: Subtraction $X - 3 = 12$

"Undo" by: Addition $X - 3 + 3 = 12 + 3$

Solution: $X = 15$

4. Similarly, multiplication and division are inverse operations. Multiplication can be used to "undo" an equation involving division, and division can be used to "undo" an equation involving multiplication. To undo an equation involving multiplication, divide both sides of the equation by the coefficient of the variable in the equation.

Operation: Multiplication $2X = 22$

"Undo" by Division $\frac{2X}{2} = \frac{22}{2}$

Solution: $X = 11$

To "undo" an equation involving division, multiply both sides of the equation by the divisor of the variable.

Operation: Division $\frac{X}{4} = 6$

"Undo" by Multiplication $\frac{4(X)}{4} = 4(6)$

Solution: $X = 24$

ACTIVITY 1 - OUTCOMES 1 and 3

Use transformations by addition, subtraction, multiplication, or division to solve equations having a variable.

Example 1: $32 = Y + 4$

$$32 - 4 = Y + 4 - 4$$

$$28 = Y$$

Example 2: $a + \frac{1}{2} = 4\frac{1}{2}$

$$a + \frac{1}{2} - \frac{1}{2} = 4\frac{1}{2} - \frac{1}{2}$$

$$a = 4$$

Example 3: $12b = 144$

$$\frac{12b}{12} = \frac{144}{12}$$

$$b = 12$$

ACTIVITY 2 - OUTCOMES 2 and 4

Use transformations by addition, subtraction, multiplication, or division with substitution and the opposite of a number to solve equations. Express the variable as a positive number.

Example 1: $-X - 6 = 4$

$$-X - 6 + 6 = 4 + 6$$

$$-X = 10$$

$$X = -10$$

Example 2: $\frac{-t}{2} = 24$

$$2\left(\frac{-t}{2}\right) = 2(24)$$

$$-t = 48$$

$$t = -48$$

Example 3: $(-r + 2) + 5 = 7$

$$-r + 7 = 7$$

$$-r + 7 - 7 = 7 - 7$$

$$-r = 0$$

ACTIVITY 3 - OUTCOMES 1, 2, and 3

Use transformations by addition, subtraction, multiplication, or division with substitution and the absolute value to express all solutions to equations involving the absolute value of the variable.

Example 1:

$$|x| + 2 = 6$$

$$|x| + 2 - 2 = 6 - 2$$

$$|x| = 4$$

$$x = +4 \text{ or } -4$$

Example 2:

$$3(|y| - 2) - |y| - 17 = 1$$

$$3|y| - 6 - |y| - 17 = 1$$

$$2|y| - 23 = 1$$

$$2|y| = 24$$

$$\frac{2|y|}{2} = \frac{24}{2}$$

$$|y| = 12$$

$$y = +12 \text{ or } -12$$

ALGEBRA I

AREA: SIMPLE EQUATIONS, INEQUALITIES, AND PROBLEMS

OUTCOMES: 5. Solve a linear equation of the form $ax - b = c$.
6. Solve a linear equation of the form $ax - b = c + dx$.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Be able to solve equations involving simple addition, subtraction, multiplication, and division.
2. Be able to apply the distributive property.

Steps to Success

1. The following procedure will be used to solve linear equations:
 - a. Simplify each member of the equation.
 - b. Use the inverse operations to undo any indicated additions or subtractions.
 - c. Use the inverse operations to undo any indicated multiplications or divisions involving the variable.
 - d. Solve and check.
2. When the variable appears in both members of the equation, transform the equation into an equivalent equation in which the variable is in only one member and solve.
3. To solve an equation containing parentheses, transform it into an equation without the parentheses. Perform the indicated operations within the parentheses first and solve the equation.

ACTIVITY 1 - OUTCOMES 5 and 6

Solving equations where variables and numbers appear on both sides of the equation, such as $8x = 60 - 4x$, can be compared to cowboys rounding up wild horses. If the $=$ sign is the fence, the cowboys want all the horses (numbers) on one side of the fence and the cowboys (variables) on the other side.

TYPE 1: EQUATIONS WITH VARIABLES IN BOTH MEMBERS

Example 1: $8x = 60 - 4x$

In this example, the $-4x$ on the right side should be eliminated. This can be done by adding $+4x$ to both members of the equation.

$$\begin{array}{r} 8x = 60 - 4x \\ + 4x \qquad + 4x \\ \hline 12x = 60 \end{array}$$

To solve the equation for the value of x , the 12 should be eliminated. This can be done by dividing both members of the equation by 12.

$$\begin{array}{r} \frac{12x}{12} = \frac{60}{12} \\ x = 5 \end{array}$$

Check:

$$\begin{array}{l} 8x = 60 - 4x \\ 8(5) = 60 - 4(5) \\ 40 = 60 - 20 \\ 40 = 40 \end{array}$$

ACTIVITY 1 - OUTCOMES 5 and 6 (Cont'd)

Example 2: $5a - 12 = 24 + 8a$

What must be eliminated on either side of the equation? (-12 from the left member and +8a from the right number)

Add + 12
to both members

$$\begin{array}{rcl} 5a - 12 & = & 24 + 8a \\ + 12 & + & 12 \\ \hline \end{array}$$

Subtract 8a
from both members

$$\begin{array}{rcl} 5a & = & 36 + 8a \\ -8a & & -8a \\ \hline -3a & = & 36 \end{array}$$

Divide both members by -3.

$$\begin{array}{rcl} -3a & = & 36 \\ -3 & & -3 \\ \hline \end{array}$$

$$a = -12$$

Check:

$$\begin{array}{l} 5a - 12 = 24 + 8a \\ 5(-12) - 12 = 24 + 8(-12) \\ -60 - 12 = 24 - 96 \\ -72 = -72 \end{array}$$

NOTE: Show students that the same answer can be derived by getting variables and numbers on opposite sides.

$$\begin{array}{rcl} 5a - 12 & = & 24 + 8a \\ - 24 & - & 24 \\ \hline 5a - 36 & = & 8a \\ - 5a & & - 5a \\ \hline - 36 & = & 3a \\ -36 & & 3a \\ \hline 3 & = & 3a \\ - 12 & = & a \end{array}$$

ACTIVITY 1 - OUTCOMES 5 and 6 (Cont'd)

TYPE 2: EQUATIONS REQUIRING THE COMBINING OF LIKE TERMS

Some equations have more than one group of variables and numbers on both sides of the equation.

Example: $6y - 12 - y = 9y + 5 + y + 3$

In this case, first combine like terms on each side.

	$(6y - y) - 12$	$=$	$(9y + y) + (5 + 3)$
	$5y - 12$	$=$	$10y + 8$
Add 12 to both members.	<u>$+ 12$</u>		<u>$+ 12$</u>

	$5y$	$=$	$10y + 20$
Subtract $10y$ from both members.	<u>$- 10y$</u>	$=$	<u>$- 10y$</u>

	$- 5y$	$=$	20
Divide both members by -5 .	<u>$- 5y$</u>	$=$	<u>20</u>
	<u>$- 5$</u>		<u>-5</u>
	y	$=$	-4

Check:

$$\begin{aligned}
 6y - 12 - y &= 9y + 5 + y + 3 \\
 6(-4) - 12 - (-4) &= 9(-4) + 5 + (-4) + 3 \\
 -24 - 12 + 4 &= -36 + 5 - 4 + 3 \\
 -32 &= -32
 \end{aligned}$$

ACTIVITY 1 - OUTCOMES 5 and 6 (Cont'd)

TYPE 3: EQUATIONS INVOLVING PARENTHESES

Some equations may have terms already grouped. It will be necessary to simplify the grouping before continuing to solve the equation.

Example 1: $5(3b - 2) = 5$

Simplify the grouping by using the distributive property: $5(3b - 2) = 15b - 10$.

Add 10
to both members.

$$\begin{array}{rcl} 15b - 10 & = & 5 \\ + 10 & + & 10 \\ \hline \end{array}$$

Divide both members by 15.

$$\begin{array}{rcl} 15b & = & 15 \\ 15b/15 & = & 15/15 \\ b & = & 1 \end{array}$$

Check:

$$\begin{array}{rcl} 5(3b - 2) & = & 5 \\ 5(3 \cdot 1 - 2) & = & 5 \\ 5(1) & = & 5 \\ 5 & = & 5 \end{array}$$

Example 2: $2(c - 3) - 17 = 13 - 3(c + 2)$

NOTE: Most algebra students making an error in finding the solution of the problem will make their error in applying the distributive property on the right side where $-3(c + 2) = -3c - 6$.

$$\begin{array}{rcl} 2(c - 3) - 17 & = & 13 - 3(c + 2) \\ 2c - 6 - 17 & = & 13 - 3c - 6 \\ 2c - 23 & = & 7 - 3c \\ + 3c + 23 & + & 23 + 3c \\ \hline 5c & = & 30 \\ \frac{5c}{5} & = & \frac{30}{5} \\ c & = & 6 \end{array}$$

ACTIVITY 1 - OUTCOMES 5 and 6 (Cont'd)

Check:

$$2(c - 3) - 17 = 13 - 3(c + 2)$$

$$2(6 - 3) - 17 = 13 - 3(6 + 2)$$

$$2(3) - 17 = 13 - 3(8)$$

$$6 - 17 = 13 - 24$$

$$- 11 = - 11$$

ALGEBRA I

AREA: SIMPLE EQUATIONS, INEQUALITIES, AND PROBLEMS

- OUTCOMES:
7. Solve a proportion in which the unknown is a numerator in one ratio and a denominator in the other ratio.
 8. Translate words into expressions and equations using variables.
 9. Represent problem situations by expressions and equations using variables.
 10. Solve word problems using algebraic equations.
 11. Evaluate formulas.
 12. Evaluate a linear expression.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Be able to translate word problems into mathematical expressions.

Steps to Success

1. The student should translate verbal expressions into algebraic equations and solve the equations.
2. The student should simplify expressions by substituting indicated values for variables and by using the correct order of operations to solve the expressions.

ACTIVITY 1 - OUTCOMES 7, 8, 9, and 10

To translate words into equations, represent the unknown quantity with a variable. State the problem algebraically by writing an equation using the given numbers and operations with the necessary symbols.

ACTIVITY 1 - OUTCOMES 7, 8, 9, and 10 (Cont'd)

Example 1: Ten times a number is 54 more than the number. Find the number.

y = unknown number

Ten times a number is 54 more than the number

$$10x \quad y = 54 + y$$

$$\begin{aligned} \text{Solve:} \quad 10y &= 54 + y \\ -y &\quad -y \\ \hline 9y &= 54 \\ 9y/9 &= 54/9 \\ y &= 6 \end{aligned}$$

$$\begin{aligned} \text{Check:} \quad 10y &= 54 + y \\ 10(6) &= 54 + 6 \\ 60 &= 60 \end{aligned}$$

Example 2: The sum of two numbers is 84. The second number is two more than the first number. Find the two numbers.

The unknown is the first number: x
The second number is represented by: $x + 2$

because the second number is two more than the first number.

Represent the statement algebraically.

First number plus second number equals sum.

$$x + x + 2 = 84$$

ACTIVITY 1 - OUTCOMES 7, 8, 9, and 10 (Cont'd)

Solve:

$$\begin{array}{rcl} x + x + 2 & = & 84 \\ 2x + 2 & = & 84 \\ \underline{- 2} & \underline{- 2} & \\ 2x & = & 82 \\ 2x/2 & = & 82/2 \\ x & = & 41 \\ x + 2 & = & 43 \end{array}$$

Check:

$$\begin{array}{rcl} x + x + 2 & = & 84 \\ 41 + 41 + 2 & = & 84 \\ 84 & = & 84 \end{array}$$

ACTIVITY 2 - OUTCOMES 11 and 12

Sometimes more than one variable is in an algebraic expression. Most of these expressions are very easy to solve because the values of all the variables necessary to simplify the expression are given.

Example 1: The formula for the area of a trapezoid is $A = 1/2h(b_1 + b_2)$. Find the area or evaluate the formula given h equals 50 inches, b_1 equals 100 inches, and b_2 equals 25 inches.

$$\begin{array}{l} A = 1/2h(b_1 + b_2) \\ A = 1/2(50)(100 + 25) \\ A = 25(125) \\ A = 3125 \text{ square inches} \end{array}$$

ACTIVITY 2 - OUTCOMES 11 and 12 (Cont'd)

There are times when an understanding of the formula or expression is unnecessary. It is important to be able to find the value of the formula or expression given the values represented by the variables.

Example 2: Evaluate the expression if $a = 2$, $b = 3$, and $x = 12$.

$$\begin{aligned} &5(a + b) - \frac{ax}{b} \\ &5(2 + 3) - \frac{2(12)}{3} \\ &5(5) - \frac{24}{3} \\ &25 - 8 \\ &17 \end{aligned}$$

ALGEBRA I

AREA: SIMPLE EQUATIONS, INEQUALITIES, AND PROBLEMS

- OUTCOMES:
13. Solve inequalities by substitution.
 14. Solve inequalities by transformations.
 15. Solve problems involving inequalities.
 16. Identify a subset of the solution set for an inequality.
 17. Find the solution sets of combined inequalities.
 18. Graph the solution sets of inequalities.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Know what is meant by the term "inequality" and the symbols $<$, $>$, $\leq$, $\geq$.
2. Be able to graph an inequality on a number line.
3. Know how to solve simple linear equations.

Steps to Success

The students should understand the following properties of inequalities:

1. The Order Property of Numbers states: If A and B are two signed numbers, then one and only one of the following is true: $A < B$, $A = B$, or $A > B$.
2. The Transitive Property states: if $A < B$ and $B < C$, then $A < C$; or if $C > B$ and $B > A$, then $C > A$.
3. The Addition Property of Inequalities states: If $a > b$, then $a + c > b + c$; and if $a < b$, then $a + c < b + c$. Simply, when the same number is added or subtracted from both members of an inequality, the order of the inequality remains unchanged.
4. The Multiplication Property of Inequalities states: If $a > b$, then $ac > bc$ when c is positive; $c > 0$. If $a < b$, then $ac < bc$ when c is positive; $c > 0$. If $a > b$, then $ac < bc$ when c is negative; $c < 0$. If $a < b$, then $ac > bc$ when c is negative; $c < 0$. Simply, when both members of an inequality are multiplied or divided by a negative number, the order of the inequality is reversed.
5. Inequalities can be solved using methods similar to those used in solving simple linear equations. Use the properties of inequalities to transform the given inequality into a simpler equivalent inequality.

ACTIVITY 1 - OUTCOMES 13, 14, 15, and 18

The only difference in solving inequalities and linear equations occurs when both sides of the inequality are divided or multiplied by a negative number. Multiplying or dividing both sides of the inequality by a negative number reverses the sign of inequality.

Example 1:

$$\begin{array}{r} 2x > 4 \\ \frac{2x}{2} > \frac{4}{2} \\ x > 2 \end{array}$$

Example 2:

$$\begin{array}{r} -2x > 4 \\ \frac{-2x}{-2} > \frac{4}{-2} \\ x < -2 \end{array}$$

Example 3:

$$\begin{array}{r} \frac{x}{6} < 3 \\ 6\left(\frac{x}{6}\right) < 6(3) \\ x < 18 \end{array}$$

Example 4:

$$\begin{array}{r} \frac{x}{-6} < 3 \\ -6\left(\frac{x}{-6}\right) < -6(3) \\ x > -18 \end{array}$$

Solve and graph the solution set of the following equation. Use the real numbers and the domain of the variables.

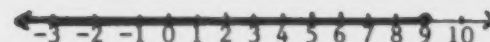
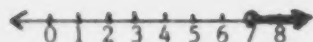
Example 5:

$$\begin{array}{r} x - 3 > 4 \\ +3 \quad +3 \\ \hline x > 7 \end{array}$$

Example 6:

$$4a + 3 \leq 39$$

$$\begin{array}{r} 4a + 3 \leq 39 \\ -3 \quad -3 \\ \hline 4a \leq 36 \\ \hline 4 \quad 4 \\ a \leq 9 \end{array}$$



ACTIVITY 1 - OUTCOMES 13, 14, 15, AND 18 (Cont'd)

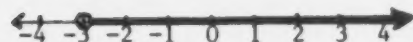
Example 7: $6b + 4 - 8b < 10$

Solve: $6b + 4 - 8b < 10$

$$-2b + 4 < 10$$

$$\begin{array}{r} -4 \quad -4 \\ \hline -2b < 6 \end{array}$$

$$\begin{array}{r} -2b \quad 6 \\ -2 \quad -2 \\ \hline b > -3 \end{array}$$



Example 8: Eight times a number is greater than 16. What numbers satisfy this condition?

Let y = unknown number.

$$\underbrace{\text{Eight}}_8 \underbrace{\text{times a number}}_x \underbrace{\text{is greater than}}_y \underbrace{\text{16}}_{16}$$

$$8y > 16$$

$$\frac{8y}{8} > \frac{16}{8}$$

$$y > 2$$



ACTIVITY 2 - OUTCOMES 16, 17, and 18

If you wanted to tell someone that you were somewhere between town A and town B, how could you state this?

I am between A and B.

A is in front of me, and B is behind me.

This could be stated in the form of symbols.

$$A < I < B \text{ or } A > I > B$$

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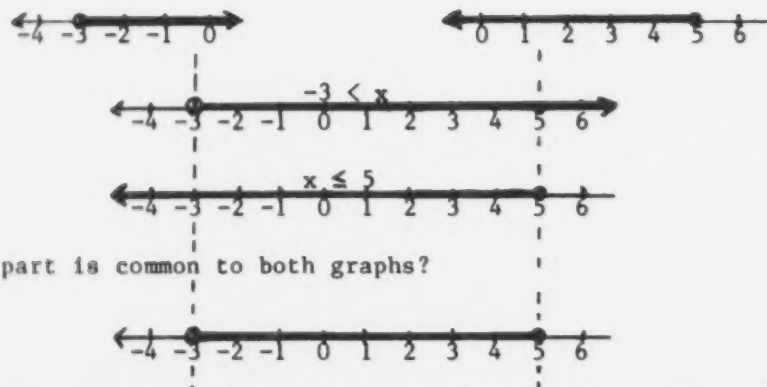
ACTIVITY 2 - OUTCOMES 16, 17, and 18 (Cont'd)

The same thing can be done to represent a value between two points on a number line.

Example 1: $-3 < x \leq 5$

$$-3 < x$$

$$x \leq 5$$



What part is common to both graphs?

The following statement is printed in the school newspaper: "Our school or our closest rival school will attend the championship game." What does this mean?

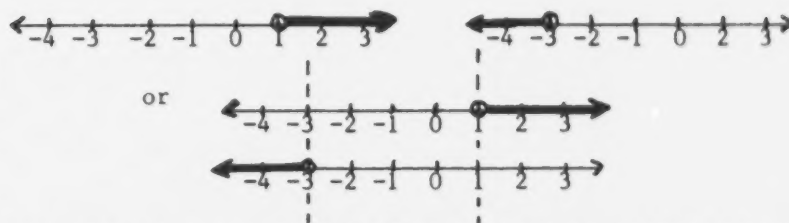
It means at least one of the schools will go to the championship game.

This information can be transferred or stated another way by using the relationship between two inequalities, i.e., one event or another event will happen.

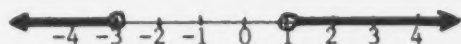
ACTIVITY 2 - OUTCOMES 16, 17, and 18 (Cont'd)

Example 2:

$$\begin{array}{rcl} 2a + 3 > 5 & \text{or} & 2a + 3 < -3 \\ 2a + 3 > 5 & & 2a + 3 < -3 \\ \underline{-3 \quad -3} & & \underline{-3 \quad -3} \\ 2a > 2 & & 2a < -6 \\ \underline{2} & & \underline{2} \\ a > 1 & \text{or} & a < -3 \end{array}$$



No common numbers exist, but the word "or" in the original statement means the solution set is at least one of the statements; therefore, the graph of the solution set would be:



The school newspaper's headline says, "Our school and our closest rival school will play in the championship game." What would the statement mean?

It means that schools would be in the game.

This statement can also be transferred into a relationship of inequalities.

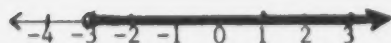
Example 3: $x - 3 < 2x$ and $3x \leq 2 + 2x$

$$\begin{array}{rcl} x - 3 < 2x & & \\ \underline{-x \quad -x} & & \\ -3 < x & & \end{array}$$

and

$$\begin{array}{rcl} 3x & \leq & 2 + 2x \\ \underline{-2x \quad -2x} & & \\ x & \leq & 2 \end{array}$$

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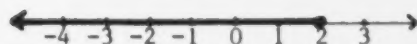
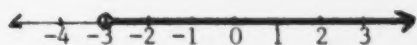


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ACTIVITY 2 - OUTCOMES 16, 17, and 18 (Cont'd)

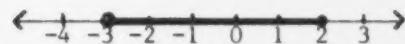
Only that which is common to both solutions satisfies the given statement because the word "and" means the points that belong to both inequalities.

The graph would be



NOTE: It may be easier for your students to see the intersection graphed like this.

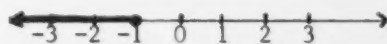
Solution:



Now do some in reverse.

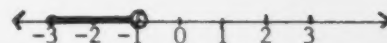
Given the solution set in the form of a graph, write the inequalities.

Example 4:



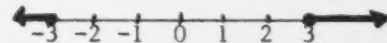
$$x \leq -1$$

Example 5:



$$-3 \leq x < -1$$

Example 6:



$$-3 \geq x \text{ or } x \geq 3$$

ALGEBRA I

AREA: WORD PROBLEMS USING ONE VARIABLE

- OUTCOMES:
1. Use vocabulary as it relates to equations.
 2. Translate words into variable expressions.
 3. Translate words into variable equations.
 4. Represent problem situations by variable expressions.
 5. Express a word problem as a number sentence.
 6. Identify the expression that can be used to solve a two-step problem.
 7. Identify the number sentence appropriate to solving a two-step problem.
 8. Solve the following types of problems: age, area, cost, digit, fraction, geometry, integers, investment, mixture, money, percent, physics, proportion, puzzle, rate-of-work, ratio, trigonometry, uniform motion, and wage.
 9. Use an algebraic method to solve word problems.

NOTES FOR THE TEACHER

Outcomes 1-7 have been covered previously in the following topics: Symbols and Numerals - Outcome 1, Use of Signed Numbers - Outcomes 4 and 5, Algebraic Vocabulary - Outcomes 1 and 2, Translation of Worded Expressions - Outcomes 3 and 4, and Writing of Word Sentences as Equations - Outcomes 9, 10 and 11.

Prerequisite for Students

1. Solve simple linear equations and inequalities.

Steps to Success

1. There are many types of word problems, but each one has the same challenge to translate the verbal relationships into algebraic equations and solve the equation.
2. It is helpful to get the students to put a heading for each word problem. The heading may be in the form of a drawing, a diagram, a chart, or in phrases stating the variable.

3. In word problems the following method should be used:

- a. Read the problem carefully.
- b. Select a variable.
- c. Write an equation.
- d. Solve the equation.
- e. Check the results.

4. The students should develop an understanding of some new terms, such as consecutive integers, consecutive even integers, consecutive odd integers, complementary angles, and supplementary angles.

ACTIVITY 1 - OUTCOMES 8 and 9

TYPE 1: INTEGER PROBLEMS

CONSECUTIVE INTEGERS

Draw a picture similar to the one below on the board or have a can labeled to represent a barrel of integers.



If I go to the barrel and pull out a number, can the number be represented algebraically without my telling you its value? Yes! Call it N .

Next, I will reach into the barrel and pull out a second number. It is one more than the first number I pulled out. Represent this number algebraically: $N + 1$.

Pull another number from the barrel. It is two more than the first number. Represent it algebraically: $N + 2$.

These are called consecutive integers and may be represented as:

- N = first consecutive integer
- $N + 1$ = second consecutive integer
- $N + 2$ = third consecutive integer

ACTIVITY 1 - OUTCOMES 8 and 9 (Cont'd)

Example: Find three consecutive integers whose sum is 42.

Represent the unknowns: $\left\{ \begin{array}{l} N = \text{first consecutive integer} \\ N + 1 = \text{second consecutive integer} \\ N + 2 = \text{third consecutive integer} \end{array} \right.$

Write the equation: $\left\{ \begin{array}{l} N + N + 1 + N + 2 = 42 \end{array} \right.$

Solve: $\left\{ \begin{array}{l} 3N + 3 = 42 \\ 3N = 39 \\ N = 13 \\ N + 1 = 14 \\ N + 2 = 15 \end{array} \right.$

Check: $\left\{ \begin{array}{l} \text{Is } 13 + 14 + 15 = 42? \\ 42 = 42 \quad \text{Yes!} \end{array} \right.$

CONSECUTIVE EVEN INTEGERS

There is a second barrel containing all even integers.



Take out one number from this barrel, but do not let the students see the number. Can the number be written algebraically? Yes! It would also be N .

Take another number from this barrel. Tell the students that it is the next even number above N . Represent this number algebraically. It would be $N + 2$.

Take a third number from the barrel. It is the next even integer above the second number. Represent this number algebraically as $N + 4$.

ACTIVITY 1 - OUTCOMES 8 and 9 (Cont'd)

We may represent the consecutive even integers as: N = first consecutive even integer;
 $N + 2$ = second consecutive even integer; and
 $N + 4$ = third consecutive even integer.

Example: Find four consecutive even integers so that the sum of the second and fourth is 116.

Represent the unknowns: $\left\{ \begin{array}{l} x = \text{first consecutive even integer} \\ x + 2 = \text{second consecutive even integer} \\ x + 4 = \text{third consecutive even integer} \\ x + 6 = \text{fourth consecutive even integer} \end{array} \right.$

Write the equation: $\left\{ \begin{array}{l} x + 2 + x + 6 = 116 \end{array} \right.$

Solve: $\left\{ \begin{array}{l} 2x + 8 = 116 \\ 2x = 108 \\ x = 54 \\ x + 2 = 56 \\ x + 4 = 58 \\ x + 6 = 60 \end{array} \right.$

Check: $\left\{ \begin{array}{l} \text{Is the sum of the second and fourth 116?} \\ 56 + 60 = 116 \\ 116 = 116 \quad \text{Yes!} \end{array} \right.$

CONSECUTIVE ODD INTEGERS

Go to a third barrel. This barrel contains all odd integers.



Choose a number from this barrel. Represent this number algebraically: What is it? Yes! It is N also.

Now, pick a second number. It is the next odd number after N . Represent it algebraically: $N + 2$.

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ACTIVITY 1 - OUTCOMES 8 and 9 (Cont'd)

NOTE: This is the point that many algebra students fail to see. Why are odd integers and even integers represented the same? There are two units between each consecutive odd integer and two units between each consecutive even integer. If two units are added to an odd integer, it gives the next odd integer. Choose another integer from this barrel. It is the second odd integer after N . Represent it algebraically: $N + 4$.

So we may also represent the consecutive odd integers as:

- N = first consecutive odd integer;
- $N + 2$ = second consecutive odd integer; and
- $N + 4$ = third consecutive odd integer.

Example: Find three consecutive odd integers so that the sum of the first and third is 9 more than the second odd integer.

Represent the unknowns

$$\left\{ \begin{array}{l} N = \text{first consecutive odd integer} \\ N + 2 = \text{second consecutive odd integer} \\ N + 4 = \text{third consecutive odd integer} \end{array} \right.$$

Write the equation:

$$\left\{ \begin{array}{l} N + N + 4 = 9 + N + 2 \end{array} \right.$$

Solve:

$$\left\{ \begin{array}{l} 2N + 4 = 9 + N \\ 2N - N = 9 - 4 \\ N = 5 \\ N + 2 = 7 \\ N + 4 = 9 \end{array} \right.$$

Check:

$$\left\{ \begin{array}{l} N + N + 4 = 9 + N + 2 \\ 5 + 9 = 9 + 5 \\ 14 = 14 \end{array} \right.$$

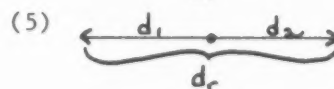
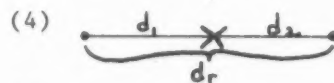
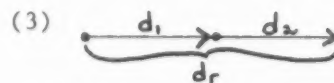
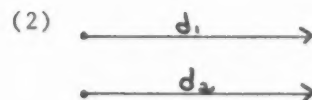
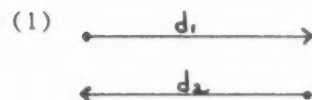
ACTIVITY 1 - OUTCOMES 8 and 9 (Cont'd)

TYPE 2: UNIFORM MOTION PROBLEMS

Every distance traveled has a rate of travel and a time of travel. This fact is represented by the formula $d = rt$.

When two modes of travel are considered, even though there are at least five different directions of motion, they can be represented in two basic equations.

Example: Let d_1 = first distance traveled
 d_2 = second distance traveled
 d_t = total distance traveled

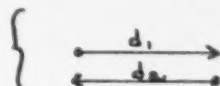


NOTE: In the first two diagrams, $d_1 = d_2$. In diagrams 3, 4, and 5, $d_1 + d_2 = d_r$. These are the basic equations for any motion problem.

ACTIVITY 1 - OUTCOMES 8 and 9 (Cont'd)

Example 1: How far can a cyclist bike at the rate of 20 mph and return over the same route at 15 mph if he only has $3\frac{1}{2}$ hours for practice?

Draw the motion:



Represent the unknowns:

$$\begin{cases} d_1 = rt = (20)(t) \\ d_2 = rt = 15(3\frac{1}{2} - t) \end{cases}$$

Write the equation:

$$\begin{cases} d_1 = d_2 \\ 20t = 15(3\frac{1}{2} - t) \\ 20t = 15(7/2 - t) \\ 20t = 105/2 - 15t \end{cases}$$

Solve:

$$\begin{cases} 35t = 105/2 \\ t = 105/70 \\ t = 1\frac{1}{2} \text{ hours} \end{cases}$$

But we are looking for the distance traveled out.

$$\begin{aligned} d_1 &= rt \\ &= (20)(t) \\ &= (20)(1\frac{1}{2}) \\ &= (20)(3/2) \\ &= 30 \text{ miles} \end{aligned}$$

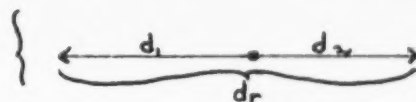
Check:

$$\begin{aligned} 20(1\frac{1}{2}) &= 15(3\frac{1}{2} - 1\frac{1}{2}) \\ 30 &= 15(2) \\ 30 &= 30 \end{aligned}$$

ACTIVITY 1 - OUTCOMES 8 and 9 (Cont'd)

Example 2: Two cars leave from the same point traveling in opposite directions. One car travels at 55 mph while the second car travels at 50 mph. In how many hours will the cars be 315 miles apart?

Draw the motion:



Represent the unknowns:

$$\begin{cases} d_1 = rt = (55)(t) \\ d_2 = rt = (50)(t) \\ d_t = 315 \text{ miles} \end{cases}$$

Write the equation:

$$\begin{cases} d_1 + d_2 = d_t \\ 55t + 50t = 315 \\ 105t = 315 \\ t = 3 \text{ hours} \end{cases}$$

Solve:

Check:

$$\begin{cases} 55(3) + 50(3) = 315 \\ 165 + 150 = 315 \\ 315 = 315 \end{cases}$$

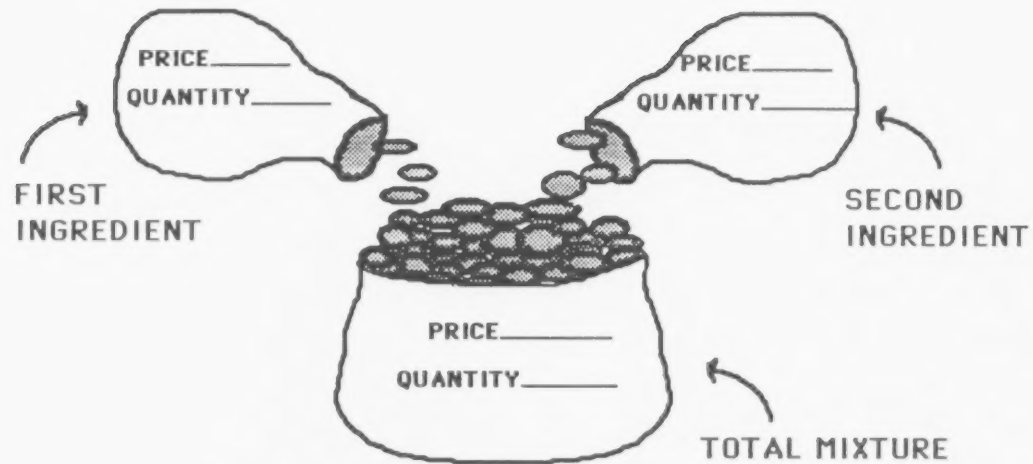
TYPE 3: MIXTURE, COST, AND MONEY PROBLEMS

Ask the students if they remember going to the store and having a fist full of change? They probably bought bubble gum, ice cream, and candy. How did they determine the cost of their purchases?

(Bubble Gum)		(Ice Cream)		(Candy)		
Price x Quantity	+	Price x Quantity	+	Price x Quantity	=	Total Cost

ACTIVITY 1 - OUTCOMES 8 and 9 (Cont'd)

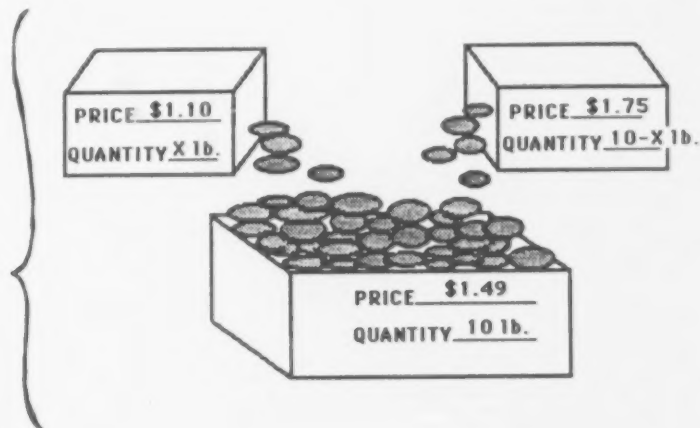
Every mixture problem can be solved in the same way regardless of what is being mixed: kinds of candy, types of nuts, gallons of gasoline, pounds of coffee, etc. Think of every mixture as shown in this picture.



ACTIVITY 1 - OUTCOMES 8 and 9 (Cont'd)

Example 1: A baker wants to make a gift box of cookies using one kind of cookies that sells for \$1.10 per pound and another kind that sells for \$1.75 per pound. How many pounds of each must be used to produce a 10-pound mixture to sell for \$1.49 a pound?

Represent the unknowns:



Write the equation:

$$1.10(x) + 1.75(10 - x) = 1.49(10)$$

Solve:

$$\begin{aligned} 1.10x + 17.50 - 1.75x &= 14.90 \\ -.65x + 17.50 &= 14.90 \\ -.65x &= -2.60 \\ x &= 4 \\ 10 - x &= 6 \end{aligned}$$

Check:

$$\begin{aligned} 1.10(4) + 1.75(6) &= 1.49(10) \\ 4.40 + 10.50 &= 14.90 \\ 14.90 &= 14.90 \end{aligned}$$

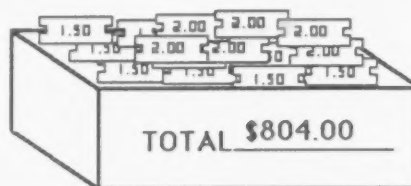
ACTIVITY 1 - OUTCOMES 8 and 9 (Cont'd)

Example 2: There were 480 tickets sold for one basketball game. The price was \$1.50 for students and \$2.00 for adults. The total receipts were \$804.00. How many students and how many adults attended the game?

Represent the unknowns:

PRICE \$1.50
STUDENTS x

PRICE \$2.00
STUDENTS 480-x



Write the equation:

$$1.50(x) + 2.00(480 - x) = 804.00$$

Solve:

$$\begin{aligned} 1.50x + 960.00 - 2.00x &= 804.00 \\ -.50x + 960.00 &= 804.00 \\ -.50x &= -156.00 \\ x &= 312 \\ 480 - x &= 168 \end{aligned}$$

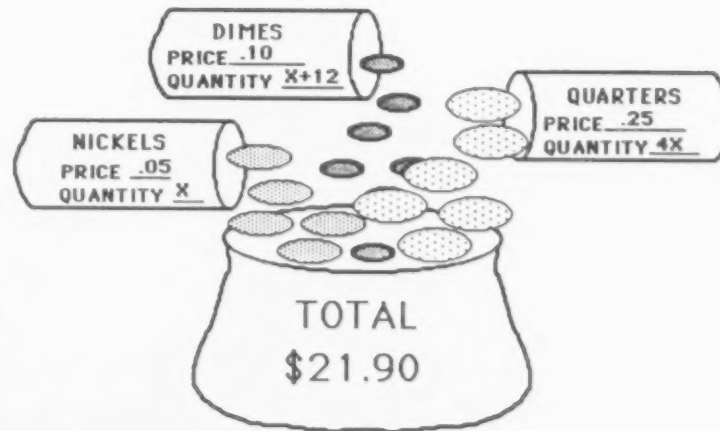
Check:

$$\begin{aligned} 1.50(312) + 2.00(168) &= 804.00 \\ 468.00 + 336.00 &= 804.00 \\ 804.00 &= 804.00 \end{aligned}$$

ACTIVITY 1 - OUTCOMES 8 and 9 (Cont'd)

Example 3: After a day of collecting on his route, a paper boy found he had a total of \$21.90. He had 12 more dimes than nickels and 4 times as many quarters as nickels. How many nickels, dimes, and quarters did he have?

Represent the unknowns:



Write the equation:

$$.05(x) + .10(x + 12) + .25(4x) = 21.90$$

Solve:

$$.05x + .10x + 1.20 + 1.00x = 21.90$$

$$1.15x + 1.20 = 21.90$$

$$1.15x = 20.70$$

$$x = 18$$

$$x + 12 = 30$$

$$4x = 72$$

Check:

$$.05(18) + .10(30) + .25(72) = 21.90$$

$$.90 + 3.00 + 18.00 = 21.90$$

$$21.90 = 21.90$$

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ACTIVITY 1 - OUTCOMES 8 and 9 (Cont'd)

TYPE 4: GEOMETRY

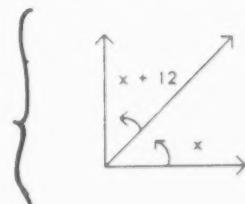
If the sum of two angles is 90° and one angle measures 30° , what is the measure of the second angle?
It is 60° because $90^\circ - 30^\circ = 60^\circ$.

If the sum of two angles is 90° and one angle measures x degrees, what is the measure of the second angle?
It is $(90 - x)$ degrees, because it cannot be simplified until the value of x is known.

The two angles whose sum is 90° are called complementary angles. If one angle is 60° , then its complement is 30° . If one angle is x degrees, then its complement is $(90 - x)$ degrees.

Example 1: An angle is 12° more than its complement. Find the degree measure of its complement.

Represent the unknowns:



Write the equation:

$$\left\{ \begin{array}{l} x + x + 12 = 90 \end{array} \right.$$

Solve:

$$\left\{ \begin{array}{l} 2x + 12 = 90 \\ 2x = 78 \\ x = 39^\circ \\ x + 12 = 51^\circ \end{array} \right.$$

Check:

$$\left\{ \begin{array}{l} \text{Does } 39^\circ + 51^\circ = 90^\circ? \\ 90^\circ = 90^\circ \end{array} \right.$$

If the sum of two angles is 180° and one angle measures 105° , what is the measure of the second angle? It is 75° , because $180^\circ - 105^\circ = 75^\circ$.

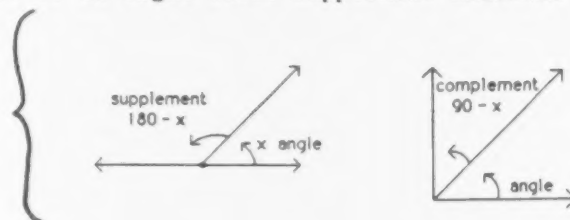
ACTIVITY 1 - OUTCOMES 8 and 9 (Cont'd)

If the sum of two angles is 180° and one angle measures x degrees, what is the measure of the second angle? It is $(180 - x)$ degrees.

The two angles whose sum is 180° are supplementary angles. If one angle is 105° , then its supplement is 75° . If one angle is x degrees, then its supplement is $(180 - x)$ degrees.

Example 2: Find the measure of an angle whose supplement measures 22° more than twice its complement.

Represent
the unknowns:



Write the
equation:

$$180 - x = 2(90 - x) + 22$$

Solve:

$$\begin{aligned} 180 - x &= 180 - 2x + 22 \\ 180 + x &= 202 \\ x &= 22^\circ \end{aligned}$$

Check:

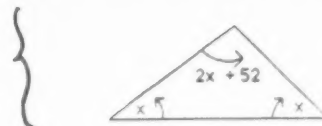
$$\begin{aligned} \text{Does } 180^\circ - 22^\circ &= 2(90^\circ - 22^\circ) + 22^\circ? \\ 158 &= 180 - 44 + 22 \\ 158^\circ &= 158^\circ \quad \text{Yes!} \end{aligned}$$

ACTIVITY 1 - OUTCOMES 8 and 9 (Cont'd)

How many degrees are in the sum of the angles of any triangle? There are 180° .

Example 3: In an isosceles triangle the third angle is 52° more than the sum of the two base angles. Find the measures of all three angles. NOTE: The base angles of an isosceles triangle are congruent.

Represent the unknowns:



Write the equation:

$$x + x + 2x + 52 = 180^\circ$$

Solve:

$$4x + 52 = 180$$

$$4x = 128$$

$$x = 32^\circ \text{ first angle}$$

$$x = 32^\circ \text{ second angle}$$

$$2x + 52 = 116^\circ \text{ third angle}$$

Check:

{ Does the sum of $32^\circ + 32^\circ + 116^\circ = 180^\circ$? Yes!

ALGEBRA I

AREA: SPECIAL PRODUCTS AND FACTORING

- OUTCOMES:
1. Use a short method to find the product of two binomials.
 2. Determine the product of a binomial to the third power.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Know the meaning of monomial, binomial, and polynomial.
2. Know how to multiply a monomial times a polynomial.
3. Know how to apply the distributive property for multiplication.

Steps to Success

1. To multiply a binomial times a binomial, use the distributive property to multiply each term in the first binomial times each term in the second binomial and combine like terms.
2. To cube a binomial, continue the process of multiplying two binomials and multiply their product by the binomial again.

ACTIVITY 1 - OUTCOME 1

You are going to be agent 00X. It is your mission to decipher the following code:

odayT si a oodg ayd.

Agent 00X, what is the message?

Today is a good day.

ACTIVITY 1 - OUTCOME 1 (Cont'd)

Since you did so well with that one, use only the letters T, S, and A to fill in the blanks to decode the following message:

_ L L _ L G E B R _ _ _ U D E N _ _ _ R E S M _ R _ _ _ U D E N _ S.

Well, Agent 00X, you are really sharp. ALL ALBEBRA STUDENTS ARE SMART STUDENTS. In algebra, try to do things the shortest way possible.

00X, your expertise is needed to develop a code that will help make multiplication of binomials as easy and as short as possible.

Recall that two binomials may be multiplied.

$$\begin{array}{r} 2x + 5 \\ 4x - 3 \\ \hline 8x^2 + 20x \\ \quad - 6x - 15 \\ \hline 8x^2 + 14x - 15 \end{array}$$

Look at the problem written like this:

$$(4x - 3)(2x + 5)$$

Step 1: What was multiplied first?

$$(4x - 3)(2x + 5)$$

The first terms in each binomial were multiplied.

Step 2: Then what was multiplied?

$$(4x - 3)(2x + 5)$$

The outer terms in each binomial were multiplied.

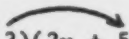
ACTIVITY 1 - OUTCOME 1 (Cont'd)

Step 3: What was multiplied next?

$$(4x - 3)(2x + 5)$$


The inner terms in each binomial were multiplied.

Step 4: What terms were multiplied next?

$$(4x - 3)(2x + 5)$$


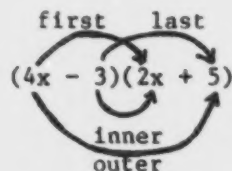
The last terms in each binomial were multiplied.

Step 5: Then what was done?

Like terms were combined.

Don't forget, Agent 00X, it is your responsibility to develop a code that will make multiplying binomials easier for everyone.

Maybe it will help to look at all steps in one diagram.



First Outer Innner Last

F O I L

ACTIVITY 1 - OUTCOME 1 (Cont'd)

A successful code for multiplication of binomials has been developed. It is called the FOIL method for multiplication.

Practice using this method until binomials can be multiplied at sight.

Example 1: $(a + 2)(a + 3)$
 $a^2 + 3a + 2a + 6$
 $a^2 + 5a + 6$

Example 2: $(2b + 3)(b - 5)$
 $2b^2 - 10b + 3b - 15$
 $2b^2 - 7b - 15$

Example 3: $(c + 8)(c - 8)$
 $c^2 - 8c + 8c - 64$
 $c^2 - 64$

Example 4: $(2d - 3e)(2d - 3e)$
 $4d^2 - 6de - 6de + 9e$
 $4d^2 - 12de + 9e$

ACTIVITY 2 - OUTCOME 2

Remember: a^1 means a
 a^2 means $a \cdot a$
 a^3 means $a \cdot a \cdot a$

The same is true for binomials.

$(a + b)^1$ means $(a + b)$
 $(a + b)^2$ means $(a + b)(a + b)$
 $(a + b)^3$ means $(a + b)(a + b)(a + b)$

ACTIVITY 2 - OUTCOME 2 (Cont'd)

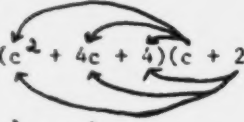
Example 1:

$$(c + 2)^3 = (c + 2)(c + 2)(c + 2) =$$

Use foil method to multiply the two binomials first:

$$(c^2 + 4c + 4)(c + 2) =$$

Use the distribution property to multiply the first term of the binomial times the polynomial:

$$(c^2 + 4c + 4)(c + 2) = c^3 + 4c^2 + 4c$$


Use the distributive property to multiply the second term of binomial times the polynomial:

$$2c^2 + 8c + 8$$

Combine like terms:

$$c^3 + 6c^2 + 12c + 8$$

Example 2:

$$\begin{array}{r} (d - 3)^3 \\ (d - 3)(d - 3)(d - 3) \\ (d^2 - 6d + 9)(d - 3) \\ d^3 - 6d^2 + 9d \\ \quad - 3d^2 + 18d - 27 \\ \hline d^3 - 9d^2 + 27d - 27 \end{array}$$

ALGEBRA I

AREA: SPECIAL PRODUCTS AND FACTORING

OUTCOME: 3: Factor: integers, monomials, polynomials, quadratic trinomials, and trinomial squares.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Be able to multiply binomials.
2. Understand primes, quadratic trinomials, and trinomial squares.
3. Understand factors for integers and monomials.

Steps to Success

1. The students' abilities to evaluate a polynomial and recognize the appropriate method to factor that type of polynomial will greatly enhance their success in factoring.
2. To factor a common monomial: (a) Find the greatest common factor. (b) Divide the polynomial by the monomial factor. (c) Write the indicated product of the two factors.
3. The difference in two squares is factored into a binomial sum and a binomial difference.
4. A trinomial square is factored into the square of a binomial.
5. A quadratic trinomial is factored by inspection and trial.

ACTIVITY 1 - OUTCOME 3

The services of Agent 00X are going to be required once again to help discover a process for factoring. There are several types of factoring. Our success in factoring will depend upon the ability to seek out or recognize the clues that will lead to the correct factors.

TYPE 1: FACTORING INTEGERS AND MONOMIALS

When two integers are multiplied, a product is found. Each of the integers multiplied is called a factor. One set of factors for 6 are 3 and 2 because $3 \times 2 = 6$. Some integers may have many factors, such as the factors of 30 are 2×15 , 3×10 , 5×6 , and $2 \times 3 \times 5$.

ACTIVITY 1 - OUTCOME 3 (Cont'd)

When an integer has no factors other than 1 and itself, it is called a prime number.

When factoring integers, it is best to start with the smallest prime factor.

Example 1: Express 350 as a product of prime factors.

Don't forget to start with the smallest prime factor each time.

$$\begin{aligned} 350 &= 2 \cdot 175 \\ &= 2 \cdot 5 \cdot 35 \\ &= 2 \cdot 5 \cdot 5 \cdot 7 \end{aligned}$$

This product can be written by a shorter method.

$$350 = 2 \cdot 5^2 \cdot 7$$

Example 2: Find the greatest common factor for the pair of integers 60, 36.

Find the prime factors of each number first.

$$\begin{aligned} 60 &= 2 \cdot 30 \\ &= 2 \cdot 2 \cdot 15 \\ &= 2 \cdot 2 \cdot 3 \cdot 5 \\ &= 2^2 \cdot 3 \cdot 5 \end{aligned} \qquad \begin{aligned} 36 &= 2 \cdot 18 \\ &= 2 \cdot 2 \cdot 9 \\ &= 2 \cdot 2 \cdot 3 \cdot 3 \\ &= 2^2 \cdot 3^2 \end{aligned}$$

Each set of factors contains a factor of 2^2 and 3. So the greatest common factor is $2^2 \cdot 3 = 4 \cdot 3 = 12$.

Example 3: Find the greatest common factor for $8a^2b$ and $18ab^3$.

$$8a^2b = 2^3 \cdot a^2 \cdot b \qquad 18ab^3 = 2 \cdot 3^2 \cdot a \cdot b^3$$

Now, look for the greatest number factor for each monomial as well as the greatest degree for each variable that is a factor of both like variables.

$$\begin{aligned} 2 \cdot a \cdot b &= \\ 2ab &\text{ is the greatest common factor.} \end{aligned}$$

ACTIVITY 1 - OUTCOME 3 (Cont'd)

TYPE 2: COMMON MONOMIAL FACTORING

The clue in common monomial factoring is to find a monomial factor that is a factor of every term of the polynomial.

Example 1: Factor $6a - 6b$.

What is the common factor? It is 6.

Divide the polynomial by the common factor.

Stress that this step
is done mentally

$$\frac{\cancel{6}a}{\cancel{6}} - \frac{\cancel{6}b}{\cancel{6}} = a - b$$

Write the indicated product of the two factors.

$$6(a - b)$$

Example 2: $10c^2d - 35cd^2$

$$5cd(2c - 7d)$$

These factors can be checked by multiplying the factors to get the original polynomial.

$$5cd(2c - 7d) = 10c^2d - 35cd^2$$

TYPE 3: FACTOR THE DIFFERENCE IN TWO SQUARES

Remember that when some binomials are multiplied, the outer term and inner term are cancelled out when they are combined.

$$\begin{array}{l} (a + b)(a - b) \\ a^2 - ab + ab - b^2 \\ a^2 - b^2 \end{array}$$

In this case the product is called the difference in two squares. The clue is two squared terms like a^2 and b^2 with a minus sign between them.

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ACTIVITY 1 - OUTCOME 3 (Cont'd)

Since the symmetric property of equality is reversible,

if $(a + b)(a - b) = a^2 - b^2$, then $a^2 - b^2 = (a + b)(a - b)$.

Example 1: Factor and check: $f^2 - 36$.

First clue: Is there a common factor? No.

Second clue: Is it the difference in two squares? Yes.

Then it will factor into a binomial sum and a binomial difference.

$$\begin{array}{l} f^2 - 36 \\ (f + 6)(f - 6) \end{array}$$

$$\begin{array}{l} \text{Check: } (f + 6)(f - 6) \\ f^2 - 6f + 6f - 36 \\ f^2 - 36 \end{array}$$

Example 2: Factor and check: $9g^2 - 81$

First clue: Is there a common monomial factor? Yes.

Factor the common monomial: $9(g^2 - 9)$.

Second clue: Is the binomial factor the difference in two squares? Yes.

Then factor the binomial factor $9(g^2 - 9)$
into a binomial sum and binomial difference: $9(g + 3)(g - 3)$.

$$\begin{array}{l} \text{Check: } 9(g + 3)(g - 3) \\ 9(g^2 - 9) \\ 9g^2 - 81 \end{array}$$

ACTIVITY 1 - OUTCOME 3 (Cont'd)

TYPE 4: FACTORING TRINOMIAL SQUARES

Remember squaring binomials? The product from squaring a binomial is a trinomial square.

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) = \\ &= a^2 + ab + ab + b^2 = \\ &= a^2 + 2ab + b^2\end{aligned}$$

Look at the product. Determine what clues will make it clear that this is a trinomial square. When all three answers are yes, this is a trinomial square which will factor into the square of a binomial.

Look at the trinomial.

Is this the square of the first term?

Is this twice the product of the two terms?

Is this the square of the second term in the binomial?

$$\begin{array}{c}(a + b)^2 \\ a^2 + 2ab + b^2\end{array}$$

Example 1: Is $h^2 - 2h + 1$ a trinomial square?

Is the first term the square of a monomial? Yes: $(h)^2$.

Is the last term the square of a monomial? Yes: $(+1)^2$ or $(-1)^2$.

Is the middle term twice the product of the two factors? Yes: $2(h)(-1) = -2h$.

So $h^2 - 2h + 1$ is a trinomial square and can be factored into $(h - 1)^2$.

ACTIVITY 1 - OUTCOME 3 (Cont'd)

Example 2: Factor $j^2 + 10j + 25$.

First clue: Is there a common monomial factor? No.
 Second clue: Is this the difference in two squares? No.
 Third clue: Is this a trinomial square? Yes.

The first term is a square: $(j)^2 = j^2$.
 The third term is a square: $(5)^2 = 25$.
 The middle term is twice the product of j and 5 : $2(5)(j) = 10j$.

Therefore, the trinomial square: $j^2 + 10j + 25$
 will factor into: $(j + 5)(j + 5)$
 or: $(j + 5)^2$

Example 3: Factor $6y^4 - 12y^2z^2 + 6z^4$

First clue: Is there a common monomial factor? Yes.
 Factor the common monomial: $6(y^4 - 2y^2z^2 + z^4)$.

Second clue: Is the remaining factor the difference in two squares? No.

Third clue: Is the trinomial factor a trinomial square? Yes.

The first term, y^4 is the square of a monomial: $(y^2)^2$.
 The third term, z^4 , is the square of a monomial: $(-z^2)^2$.
 The middle term, $-2y^2z^2$, is twice the product of these: $2(y^2)(-z^2) = -2y^2z^2$.

Therefore, the trinomial square: $6(y^4 - 2y^2z^2 + z^4)$
 will factor into: $6(y^2 - z^2)^2$

The fourth clue: There has been something like this binomial factor before. It was the difference in two squares.

Factor again: $6(y^2 - z^2)^2$
 $6[(y + z)(y - z)]^2$
 $6(y + z)^2(y - z)^2$

ACTIVITY 1 - OUTCOME 3 (Cont'd)

TYPE 5: FACTORING QUADRATIC TRINOMIALS

This might be called the hunt-and-see method or trial-and-error method. First, check out the possibilities. Then by trial and error or good judgement, select the correct combination of factors.

First, look at the clues that can be gathered by looking at the signs as they appear in a trinomial.

Case 1: $ax^2 + bx + c$

Clue: When the signs in the trinomial are both positive, there is only one possibility for the signs in the binomial factors.

$$(\_ + \_)(\_ + \_)$$

Case 2: $ax^2 - bx + c$

Clue: When the middle term in the trinomial is negative and the third term is positive, there is only one possibility for the signs of the factors:

$$(\_ - \_)(\_ - \_)$$

Case 3: $ax^2 - bx - c$

Clue: When both signs are negative in the trinomial, then one sign will be positive and one sign will be negative.

$$(\_ + \_)(\_ - \_)$$

In this case the largest factor of the third term will be negative.

Case 4: $ax^2 + bx - c$

Clue: When the middle term of the trinomial is positive and the third term is negative, the signs will also be one positive and one negative.

$$(\_ + \_)(\_ - \_)$$

In this case, however, the largest factor of the third term will be positive.

ACTIVITY 1 - OUTCOME 3 (Cont'd)

By checking these clues, the likelihood of making a mistake in factoring the given trinomial can be reduced.

Example 1: Factor $K^2 + 5K + 6$ and check.

First clue: Does it have a common monomial factor? No.

Second clue: Is it the difference in two squares? No.

Third clue: Is it a trinomial square? No.

Fourth clue: Is it a quadratic trinomial? Yes.

$$K^2 + 5K + 6$$

Check the signs and determine the signs of the factors.

$$(+) (+)$$

Choose factors of the first term.

$$(K +)(K +)$$

Choose factors of the third term.

$$\begin{array}{ll} (K + 1)(K + 6) & \text{(This may be} \\ \text{or } (K + 2)(K + 3) & \text{done mentally.)} \end{array}$$

Choose the set of factors whose sum of the outer and inner terms equals the middle term of the trinomial.

$$2K + 3K = 5K \quad \text{therefore} \quad (K + 2)(K + 3) \text{ are the correct factors.}$$

To check, multiply the factors:

$$\begin{array}{l} (K + 2)(K + 3) \\ K^2 + 2K + 3K + 6 \\ K^2 + 5K + 6 \end{array}$$

ACTIVITY 1 - OUTCOME 3 (Cont'd)

Example 2: Factor $5m^2 - 7m + 2$ and check.

First clue: Does it have a common monomial factor? No.
Second clue: Is it the difference in two squares? No.
Third clue: Is it a trinomial square? No.
Fourth clue: Is it a quadratic trinomial? Yes.

$$5m^2 - 7m + 2$$

Look at the signs and determine the signs of the factors.

$$(-)(-)$$

Choose the possible factors of the first term.

$$(5m -)(m -)$$

or $(m -)(5m -)$

Choose the possible factors of the third term.

$$(5m - 1)(m - 2)$$

or $(m - 1)(5m - 2)$

Check the outer and inner terms of each set of factors. Choose the set of factors whose sum of the products of the outer and inner terms equals the middle term of the trinomial.

$$- 5m - 2m = - 7m$$

and $(m - 1)(5m - 2)$ are the factors.

To check:

$$\begin{array}{l} (m - 1)(5m - 2) \\ 5m^2 - 2m - 5m + 2 \\ 5m^2 - 7m + 2 \end{array}$$

NOTE: There is a game for practice described in the Curriculum Guide for Algebra II called "Old Poly."

ALGEBRA I

AREA: SPECIAL PRODUCTS AND FACTORING

OUTCOME: 4: Use factoring to determine the solution sets of quadratic equations.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Solve simple linear equations.
2. Be able to factor polynomials.
3. Know that every positive real number has two real square roots - one positive and one negative.
4. Remember the product property for zero:

$$ab = 0 \text{ if } a = 0 \text{ or } b = 0.$$

Steps to Success

1. The student's ability to recognize a quadratic equation will initiate the solving process.
2. The following method for solving quadratic equations will be used:
 - a. Transform the equation into standard form: $ax^2 + bx + c = 0$.
 - b. Factor the left number.
 - c. Set each factor equal to zero.
 - d. Solve the resulting linear equations.

ACTIVITY 1 - OUTCOME 4

Look at the equation $x^2 - 3x - 10 = 0$. It is different from the other equations that have been solved.

One thing in this equation has not been seen in any equation. What is it? x^2

This equation is of degree two because the highest degree term is two. It is called a quadratic equation.

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ACTIVITY 1 - OUTCOME 4 (Cont'd)

Students must be able to solve a quadratic equation, but they cannot simply combine like terms and simplify as they did in linear equations. A hint to its solution is the product property for zero. If $ab = 0$ then $a = 0$ or $b = 0$. Can the equation $x^2 - 3x - 10 = 0$ be written as a product? Yes, the left member can be factored.

$$\begin{aligned}x^2 - 3x - 10 &= 0 \\(x - 5)(x + 2) &= 0\end{aligned}$$

Now set each factor equal to zero.

$$\begin{aligned}x^2 - 3x - 10 &= 0 \\(x - 5)(x + 2) &= 0 \\x - 5 = 0 \quad x + 2 &= 0\end{aligned}$$

Solve the resulting linear equations.

$$\begin{aligned}x - 5 &= 0 & x + 2 &= 0 \\x &= 5 & x &= -2\end{aligned}$$

There are two solutions: $x = 5$ or $x = -2$. Can there be two solutions? Can both be correct solutions?

Check by substitution.

$\begin{aligned}\text{if } x &= 5 \\x^2 - 3x - 10 &= 0 \\(5)^2 - 3(5) - 10 &= 0 \\25 - 15 - 10 &= 0 \\0 &= 0\end{aligned}$	$\begin{aligned}\text{if } x &= -2 \\x^2 - 3x - 10 &= 0 \\(-2)^2 - 3(-2) - 10 &= 0 \\4 + 6 - 10 &= 0 \\0 &= 0\end{aligned}$
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Yes, both are correct solutions; therefore, $x = \{5, -2\}$.

Thus, one way to solve quadratic equations is by factoring.

ACTIVITY 1 - OUTCOME 4 (Cont'd)

Example 1: Solve $R^2 - 36 = 0$

Write the equation
in standard form:
Factor left member:

$$R^2 - 36 = 0$$

$$(R + 6)(R - 6) = 0$$

Set each factor equal to 0:

$$R + 6 = 0 \quad R - 6 = 0$$

Solve the resulting equations:

$$\begin{array}{ll} R + 6 = 0 & R - 6 = 0 \\ R = -6 & R = 6 \end{array}$$

Check:

$$\begin{array}{ll} R^2 - 36 = 0 & R^2 - 36 = 0 \\ (-6)^2 - 36 = 0 & (6)^2 - 36 = 0 \\ 36 - 36 = 0 & 36 - 36 = 0 \\ 0 = 0 & 0 = 0 \end{array}$$

Write the solution set:

$$R = \{-6, 6\}$$

Example 2: Solve $-6s = -9s^2 - 1$

Write the equation in standard form:

$$9s^2 - 6s + 1 = 0$$

Factor left member:

$$(3s - 1)(3s - 1) = 0$$

Set each factor equal to 0:

$$3s - 1 = 0$$

$$3s = 1$$

Solve the resulting equation:

$$s = 1/3$$

NOTE: This is a double root and only has to be written one time.

Check:

$$\begin{array}{l} -6s = -9s^2 - 1 \\ -6(1/3) = -9(1/3)^2 - 1 \\ -2 = -9(1/9) - 1 \\ -2 = -1 - 1 \\ -2 = -2 \end{array}$$

Write the solution set:

$$s = \{1/3\}$$

ALGEBRA I

AREA: OPERATIONS WITH FRACTIONS

- OUTCOMES:
1. Review arithmetic fractions.
 2. Write equivalent algebraic fractions.
 3. Simplify algebraic fractions.
 4. Multiply algebraic fractions.
 5. Divide algebraic fractions.
 6. Express fractions with their least common denominators.
 7. Add and subtract algebraic fractions.
 8. Write mixed expressions as a fraction in simplest form.
 9. Simplify complex fractions.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Know how to add, subtract, multiply, and divide fractions.
2. Be able to factor single and multiple term expressions.

Steps to Success

1. The students' abilities to relate their knowledge of fractions in arithmetic to algebraic fractions will determine their success.
2. The students must be reminded that division by zero cannot be performed. The value of a fraction whose denominator is zero is undefined.

3. The following rules for addition, subtraction, multiplication, and division of fractions will be used.

$$\text{Addition: } \frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$$

$$\text{Subtraction: } \frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}$$

$$\text{Multiplication: } \frac{a}{c} \cdot \frac{b}{d} = \frac{ab}{cd}$$

$$\text{Division: } \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

ACTIVITY 1 - OUTCOMES 1, 2, and 3

Fractions in algebra also are used as symbols to indicate division. Just as $5/6$ means to divide 5 by 6, a/b means to divide a by b . Algebraic fractions may look like the following examples:

$$\frac{1}{a}, \frac{t}{5}, \frac{a^2 - b^2}{c}, \frac{m - n}{p + q}, \frac{a^2 + 2ab + b^2}{a^2 - b^2}$$

Fractions can be used in various operations. Before adding, subtracting, multiplying, and dividing algebraic fractions, review the fundamentals of arithmetic fractions. In arithmetic fractions, the numerator and denominator of a fraction may be multiplied or divided by the same expression without changing the value of the fraction.

$$\text{Example: } \frac{12}{8} \cdot \frac{2}{2} = \frac{24}{16} \quad \text{or} \quad \frac{12}{8} = \frac{12 \div 4}{8 \div 4} = \frac{3}{2}$$

This is helpful when reducing a fraction to its lowest terms. Remember: a fraction is in its lowest terms when the numerator and denominator contain no common factor. $\frac{35}{55} = \frac{5(7)}{5(11)} = \frac{7}{11}$

Now, use the same principle to reduce algebraic fractions to their lowest terms. Since there may be more than one number in an algebraic fraction, remember two things:

1. When the numerator and denominator are monomials, divide both numerator and denominator by their greatest common factor.
2. When the numerator and denominator are polynomials, factor the numerator and denominator and divide by the common factor(s).

ACTIVITY 1 - OUTCOMES 1, 2, and 3 (Cont'd)

Reduce the following fractions to their lowest terms:

Example 1: $\frac{16}{8} =$

$$\frac{16 \div 8}{8 \div 8} =$$

$$\frac{2}{1} =$$

$$2$$

Example 2:

$$\frac{20x^2y}{4xy} =$$

$$\frac{20x^2y \div 4xy}{4xy \div 4xy} =$$

$$\frac{5x}{1} =$$

$$5x$$

Example 3: $\frac{2a + 2b}{5a + 5b} =$

$$\frac{2(a + b)}{5(a + b)} =$$

$$\frac{2}{5}$$

Example 4:

$$\frac{3t^2 - 9t}{t^2 - 5t + 6} =$$

$$\frac{3t(t - 3)}{(t - 3)(t - 2)} =$$

$$\frac{3t}{t - 2}$$

NOTE: Algebra students may tend to cancel any like variable. Stress that even like variables must be factors of both the numerator and denominator before they can be cancelled.

ACTIVITY 2 - OUTCOMES 4, 5, 6, and 7

MULTIPLICATION OF ALGEBRAIC FRACTIONS

To multiply algebraic fractions, multiply the numerator of the first fraction times the numerator of the second fraction and the denominator of the first fraction times the denominator of the second fraction. Before multiplying the fractions, it is advantageous to factor the numerator and denominator when possible. This reduces the product to its lowest terms.

Find the products. Reduce all answers to lowest terms.

Example 1: $\frac{1}{4} \cdot \frac{5}{6} =$

$$\frac{1 \cdot 5}{4 \cdot 6} =$$

$$\frac{5}{24}$$

Example 2: $\frac{-6ab}{5} \cdot \frac{-10a}{3a^2b} =$

$$\frac{(-6ab)(-10a)}{5(3a^2b)} =$$

$$\frac{\overset{-2}{\cancel{6}}\overset{-2}{\cancel{a}}\overset{-2}{\cancel{b}}(-10a)}{\cancel{5}(\cancel{3}\overset{-2}{\cancel{a}}\overset{-2}{\cancel{b}})} =$$

$$\frac{4}{1} =$$

$$4$$

Example 3: $\frac{(c+2)}{3} \cdot \frac{(c+2)}{4} =$

$$\frac{(c+2)(c+2)}{3 \cdot 4} =$$

$$\frac{c^2 + 4c + 4}{12}$$

Example 4: $\frac{a^2 - 9a + 18}{a^2 + 7a + 12} \cdot \frac{a^2 - a - 20}{a^2 - 7a + 10} =$

$$\frac{(a-3)(a-6)}{\cancel{(a+4)}(a+3)} \cdot \frac{\cancel{(a+4)}(a-5)}{\cancel{(a-5)}(a-2)} =$$

$$\frac{(a-3)(a-6)}{(a+3)(a-2)} =$$

$$\frac{a^2 - 9a + 18}{a^2 + a - 6}$$

ACTIVITY 2 - OUTCOMES 4, 5, 6, and 7 (Cont'd)

DIVISION OF ALGEBRAIC FRACTIONS

To divide algebraic fractions, simply invert the divisor and multiply as explained in Examples 1-4.

Example 5: $\frac{a}{b^2} \div \frac{a^2}{b} =$

$$\frac{a}{b^2} \cdot \frac{b}{a^2} =$$

$$\frac{\cancel{a} \cdot \cancel{b}}{b^2 \cdot \cancel{a^2}} =$$

$$\frac{1}{ab}$$

Example 6: $\frac{4m^2 - 8m + 3}{2m^2 - 5m + 3} \div \frac{4m^2 - 1}{3m^2 - 3m} =$

$$\frac{4m^2 - 8m + 3}{2m^2 - 5m + 3} \cdot \frac{3m^2 - 3m}{4m^2 - 1} =$$

$$\frac{(2m-1)(2m-3)}{(2m-3)(m-1)} \cdot \frac{3m(m-1)}{(2m+1)(2m-1)} =$$

$$\frac{3m}{2m+1}$$

NOTE: Many algebra students invert the wrong fraction when dividing. Stress that the divisor always follows the division sign if it is the form $a/b \div b/c$.

ADDITION AND SUBTRACTION OF LIKE ALGEBRAIC FRACTIONS

Like fractions can be added or subtracted. Fractions are alike if they have the same denominators.

To add or subtract like fractions, simply add or subtract the numerators. Reduce the fraction to lowest terms by factoring the numerator and denominator when possible and cancelling the common factors.

Example 7: $\frac{5}{13} + \frac{2}{13} =$

$$\frac{5+2}{13} =$$

$$\frac{7}{13}$$

Example 8: $\frac{3}{a+b} - \frac{2}{a+b} =$

$$\frac{3-2}{a+b} =$$

$$\frac{1}{a+b}$$

ACTIVITY 2 - OUTCOMES 4, 5, 6, and 7 (Cont'd)

Example 9: $\frac{R^2 + R}{R^2 - 9} + \frac{R - 3}{R^2 - 9} =$

$$\frac{R^2 + R + R - 3}{R^2 - 9} =$$

$$\frac{R^2 + 2R - 3}{R^2 - 9} =$$

$$\frac{(R + 3)(R - 1)}{(R + 3)(R - 3)} =$$

$$\frac{R - 1}{R - 3}$$

ADDITION AND SUBTRACTION OF UNLIKE ALGEBRAIC FRACTIONS

Add or subtract unlike fractions. Find their common denominator and express the fractions with equal (like) denominators.

Example 10: $\frac{3}{8} + \frac{5}{24} =$

$$\frac{3(3)}{3(8)} + \frac{1(5)}{1(24)} =$$

$$\frac{9}{24} + \frac{5}{24} =$$

$$\frac{9 + 5}{24} =$$

$$\frac{14}{24} =$$

$$\frac{7}{12}$$

Example 11: $\frac{2}{b} + \frac{1}{5} =$

$$\frac{5(2)}{5b} + \frac{b(1)}{5b} =$$

$$\frac{10}{5b} + \frac{b}{5b} =$$

$$\frac{10 + b}{5b}$$

ACTIVITY 2 - OUTCOMES 4, 5, 6, and 7 (Cont'd)

Example 12: $\frac{3c-2}{4} + \frac{2c-1}{6} =$

$$\frac{3(3c-2)}{12} + \frac{2(2c-1)}{12} =$$

$$\frac{9c-6}{12} + \frac{4c-2}{12} =$$

$$\frac{9c-6+4c-2}{12} =$$

$$\frac{13c-8}{12}$$

Example 13: $\frac{2}{m^2-n^2} - \frac{3}{m+n} =$

$$\frac{2}{(m+n)(m-n)} - \frac{3}{m+n} =$$

$$\frac{1(2)}{(m+n)(m-n)} - \frac{(m-n)(3)}{(m+n)(m-n)} =$$

$$\frac{2-3(m-n)}{(m+n)(m-n)} =$$

$$\frac{2-3m+3n}{(m+n)(m-n)} =$$

ACTIVITY 3 - OUTCOMES 8 and 9

MIXED EXPRESSIONS

In arithmetic the fraction $2\frac{1}{3}$ is read "2 and $\frac{1}{3}$." It means $2 + \frac{1}{3}$. These are called mixed numbers. In algebra there are also algebraic mixed numbers, which are simply a combination of whole numbers and fractions.

ACTIVITY 3 - OUTCOMES 8 and 9 (Cont'd)

Write each mixed expression as a fraction in simplest form.

Example 1: $3 + \frac{4}{5} =$

$$\frac{3}{1} + \frac{4}{5} =$$

$$\frac{5(3)}{5} + \frac{1(4)}{5} =$$

$$\frac{15}{5} + \frac{4}{5} =$$

$$\frac{19}{5}$$

Example 2: $x + \frac{3}{y} =$

$$\frac{x}{1} + \frac{3}{y} =$$

$$\frac{y(x)}{y} + \frac{1(3)}{y} =$$

$$\frac{xy + 3}{y}$$

Example 3: $a + \frac{2b}{a+b} =$

$$\frac{a}{1} + \frac{2b}{a+b} =$$

$$\frac{(a+b)(a)}{a+b} + \frac{2b}{a+b} =$$

$$\frac{a^2 + ab}{a+b} + \frac{2b}{a+b} =$$

$$\frac{a^2 + ab + 2b}{a+b} =$$

Write each fraction as a mixed expression.

Example 4: $\frac{15}{7} =$

$$7 \overline{)15} \begin{array}{r} 2 \\ 14 \\ \hline 1 \end{array} =$$

$$2 \frac{1}{7}$$

Example 5: $\frac{12 + 18c^2}{6c} =$

$$\frac{12}{6c} + \frac{18c^2}{6c} =$$

$$\frac{2}{c} + 3c$$

Example 6: $\frac{9d^2 - 6d + 5}{3d - 2} =$

$$3d - 2 \overline{)9d^2 - 6d + 5} \begin{array}{r} 3d \\ 9d^2 - 6d \\ \hline 5 \end{array} =$$

$$3d + \frac{5}{3d - 2}$$

ACTIVITY 3 - OUTCOMES 8 and 9 (Cont'd)

COMPLEX FRACTIONS

In algebra there are fractions whose numerator or denominator or both have fractions in them. These fractions are called complex fractions. One way to simplify complex fractions is to multiply both the numerator and the denominator of the complex fraction by a common denominator of the fractions.

$$\text{Example 7: } \frac{\frac{4}{5}}{\frac{6}{10}} =$$

$$\frac{10(\frac{4}{5})}{10(\frac{6}{10})} =$$

$$\frac{8}{6} =$$

$$1\frac{2}{6} =$$

$$1\frac{1}{3}$$

$$\text{Example 8: } \frac{\frac{18x^2}{5xy^2}}{\frac{9xy}{25y^2}} =$$

$$\frac{25xy^2(\frac{18x^2}{5xy^2})}{25xy^2(\frac{9xy}{25y^2})} =$$

$$\frac{5(18x^2)}{9x^2y} =$$

$$\frac{90x^2}{9x^2y} =$$

$$\frac{10}{y}$$

$$\text{Example 9: } \frac{a - \frac{2}{a+1}}{a + \frac{a-3}{a+1}} =$$

$$\frac{(a+1)\left[a - \frac{2}{a+1}\right]}{(a+1)\left[a + \frac{a-3}{a+1}\right]} =$$

$$\frac{a^2 + a - 2}{a^2 + a + a - 3} =$$

$$\frac{a^2 + a - 2}{a^2 + 2a - 3} =$$

$$\frac{(a+2)(\cancel{a-1})}{(a+3)(\cancel{a-1})} =$$

$$\frac{a+2}{a+3}$$

ALGEBRA I

AREA: ALGEBRAIC SENTENCES INVOLVING FRACTIONS

- OUTCOMES:
1. Solve fractional equations.
 2. Solve fractional inequalities.
 3. Graph solution sets of fractional inequalities.
 4. Solve word problems involving fractional equations and inequalities.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Find least common denominators.
2. Solve and graph linear equations and inequalities.
3. Solve word problems involving one and two variables.

Steps to Success

1. Know that fractional equations may be simplified in more than one way. The method to be used is to multiply by a least common denominator and solve the equation.
2. Use all processes learned in solving linear and quadratic equations and inequalities and extend these concepts to fractions.

ACTIVITY 1 - OUTCOMES 1, 2, 3, and 4

Many equations and inequalities contain fractions. In most cases fractions should be eliminated first before solving the equation.

ACTIVITY 1 - OUTCOMES 1, 2, 3, and 4 (Cont'd)

To solve:

$$\frac{x}{3} + \frac{x}{15} = 2$$

Multiply each
term by a common
denominator:

$$15\left(\frac{x}{3}\right) + 15\left(\frac{x}{15}\right) = 15(2)$$

Solve the
equation:

$$\begin{aligned} 5x + x &= 30 \\ 6x &= 30 \\ x &= 5 \end{aligned}$$

Check:

$$\begin{aligned} \frac{x}{3} + \frac{x}{15} &= 2 \\ \frac{5}{3} + \frac{5}{15} &= 2 \\ \frac{25}{15} + \frac{5}{15} &= 2 \\ \frac{30}{15} &= 2 \\ 2 &= 2 \end{aligned}$$

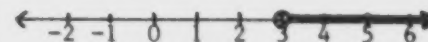
NOTE: Stress to the students these steps: (1) multiply every term on both sides of the equation by the common denominator; (2) solve each equation or inequality; (3) if the sentence is an inequality, then graph its solution set.

Example 1:

$$\begin{aligned} \frac{x+3}{2} &= \frac{27}{9} \\ 18\left(\frac{x+3}{2}\right) &= 18\left(\frac{27}{9}\right) \\ 9(x+3) &= 2(27) \\ 9x+27 &= 54 \\ 9x &= 27 \\ x &= 3 \end{aligned}$$

Example 2:

$$\begin{aligned} \frac{4a}{5} - \frac{3a}{10} &> \frac{3}{2} \\ 10\left(\frac{4a}{5}\right) - 10\left(\frac{3a}{10}\right) &> 10\left(\frac{3}{2}\right) \\ 2(4a) - 1(3a) &> 5(3) \\ 8a - 3a &> 15 \\ 5a &> 15 \\ a &> 3 \end{aligned}$$



ACTIVITY 1 - OUTCOMES 1, 2, 3, and 4 (Cont'd)

Example 1: (Cont'd)

Check: $\frac{x+3}{2} = \frac{27}{9}$
 $\frac{3+3}{2} = \frac{27}{9}$
 $\frac{6}{2} = \frac{27}{9}$
 $3 = 3$

Example 3 $\frac{2}{3x} = \frac{2}{x+4}$

$$(3x)(x+4)\left(\frac{2}{3x}\right) = (3x)(x+4)\left(\frac{2}{x+4}\right)$$

$$(x+4)(2) = (3x)(2)$$

$$2x+8 = 6x$$

$$8 = 4x$$

$$2 = x$$

Check the solution.

The solution set is {2}.

Example 4: $\frac{3y-1}{y+3} + 3 = \frac{4y}{y-3}$

$$(y+3)(y-3)\left(\frac{3y-1}{y+3}\right) + (y+3)(y-3)(3) = (y+3)(y-3)\left(\frac{4y}{y-3}\right)$$

$$(y-3)(3y-1) + (y^2-9)(3) = (y+3)(4y)$$

$$3y^2 - 10y + 3 + 3y^2 - 27 = 4y^2 + 12y$$

$$2y^2 - 22y - 24 = 0$$

Solve the quadratic equation by factoring.

$$2y^2 - 22y - 24 = 0$$

$$2(y^2 - 11y - 12) = 0$$

$$2(y-12)(y+1) = 0$$

$$y-12 = 0 \text{ or } y+1 = 0$$

$$y = 12 \quad y = -1$$

Check the solutions. The solution set is {12, -1}.

ACTIVITY 2 - OUTCOME 4

Solving word problems involving fractional equations and inequalities will be worked like the previous word problems. The only additional step will be first to eliminate the fractions from the equation.

Example 1: The larger of two numbers is 5 more than 4 times the smaller number. The smaller number is one-fifth the larger number. What are the two numbers?

Represent the unknowns: $\left\{ \begin{array}{l} x = \text{smallest number} \\ 4x + 5 = \text{larger number} \end{array} \right.$

Write the equation: $\frac{\text{Smaller number}}{x} = \frac{\text{one-fifth the larger}}{1/5(4x + 5)}$

Solve: $\left\{ \begin{array}{l} 5(x) = 5(1/5)(4x + 5) \\ 5x = 4x + 5 \\ x = 5 \\ 4x + 5 = 25 \end{array} \right.$

Check: $\left\{ \begin{array}{l} \text{Is 25 five more than 4 times 5?} \\ 25 = 5 + 4(5) \\ 25 = 25 \end{array} \right.$

Example 2: When working alone, Sue can clean her room in 20 minutes. Her sister Joan can clean the room in 30 minutes. If both worked together, how long will it take them to complete the job?

Represent the unknowns: $\left\{ \begin{array}{l} x = \text{number of minutes for both girls to do the job} \\ x/20 = \text{fraction part of job Sue can do} \\ x/20 = \text{fraction part of job Joan can do} \end{array} \right.$

Write the equation: $\left\{ \begin{array}{l} \frac{x}{20} + \frac{x}{30} = 1 \end{array} \right. \quad 1 \text{ represents the total job}$

ACTIVITY 2 - OUTCOME 4 (Cont'd)

$$\begin{array}{l} \text{Solve:} \\ \left\{ \begin{array}{l} 60\left(\frac{x}{20}\right) + 60\left(\frac{x}{30}\right) = 60(1) \\ 3x + 2x = 60 \\ 5x = 60 \\ x = 12 \end{array} \right. \end{array}$$

Therefore, both girls could clean the room in 12 minutes.

$$\begin{array}{l} \text{Check:} \\ \left\{ \begin{array}{l} \frac{x}{20} + \frac{x}{30} = 1 \\ \frac{12}{20} + \frac{12}{30} = 1 \\ \frac{36}{60} + \frac{24}{60} = 1 \\ \frac{60}{60} = 1 \\ 1 = 1 \end{array} \right. \end{array}$$

ALGEBRA I

AREA: FIRST DEGREE EQUATIONS AND INEQUALITIES CONTAINING TWO VARIABLES

- OUTCOMES:
1. Locate a point on a grid, given the values of x and y .
 2. Name the ordered pair for a point on a grid.
 3. Locate a point on a coordinate axis, given its coordinates.
 4. Identify the function that describes a given ordered pair.
 5. Identify the ordered pair that represents a solution for a graphed function.

NOTES FOR THE TEACHER

Prerequisites for Students

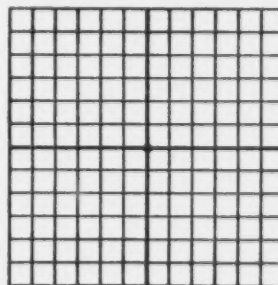
1. Recognize and understand signed numbers.
2. Graph numbers on a number line.

Step to Success

1. The students should transfer the knowledge of straight line plotting to rectangular grid plotting.

ACTIVITY 1 - OUTCOMES 1, 2, and 3

The following is a diagram of a city laid out in perfect square blocks. The address of each intersection is given by how far to the left or right and how far up or down one must travel from the center intersection, which is called the origin, and can be represented by the ordered pair $(0,0)$.

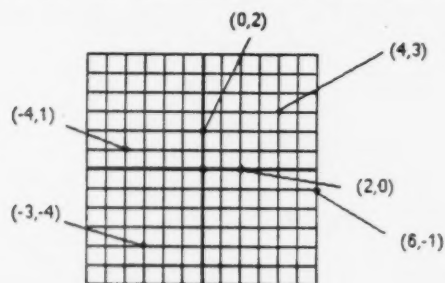


ACTIVITY 1 - OUTCOMES 1, 2, and 3 (Cont'd)

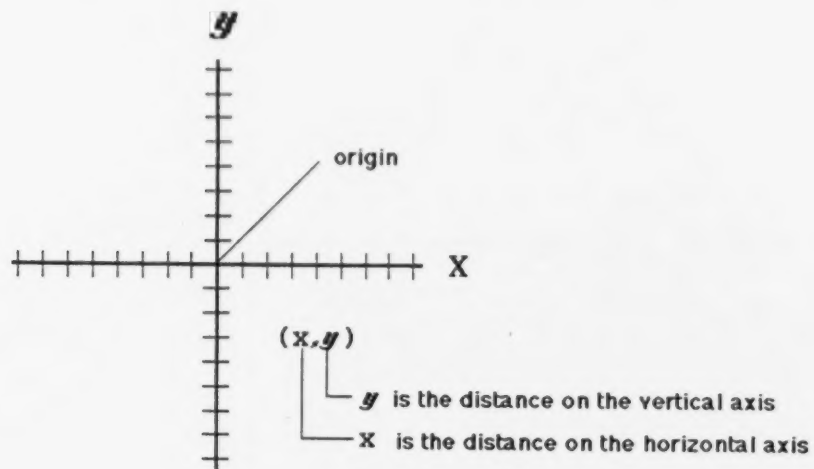
The first number in the ordered pair is how far they have traveled to the left or right, and the second number is how far they have traveled up or down. Travel to the right is a positive direction (+) and to the left is a negative direction (-). Similarly, up is positive (+), and down is negative (-).

Find the addresses for $(6, -1)$, $(-4, 1)$, $(2, 0)$, $(0, 2)$, $(-3, -4)$, and $(4, 3)$.

Place a dot on the intersection for each ordered pair.

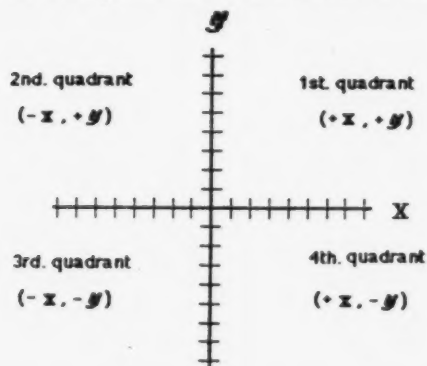


The rectangular grid is similar to a coordinate plane.

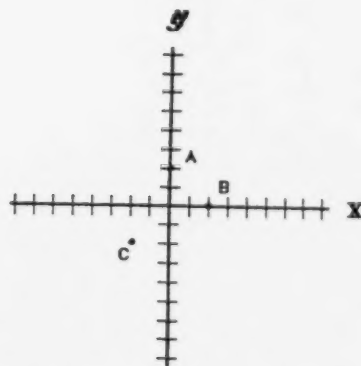


ACTIVITY 1 - OUTCOMES 1, 2, and 3 (Cont'd)

Notice the plane has been divided into 4 quadrants.



Example 1: Write the ordered pairs represented on this coordinate plane.

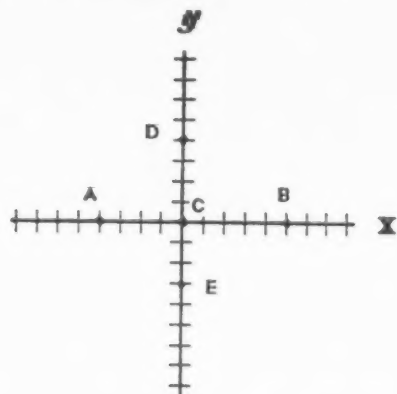


$A = (1, 2)$, $B = (2, 0)$, and $C = (-2, -2)$

ACTIVITY 1 - OUTCOMES 1, 2, and 3 (Cont'd)

Example 2: Plot the ordered pairs.

$A = (-4, 0)$, $B = (5, 1)$, $C = (0, 0)$, $D = (-1, 4)$, and $E = (2, -3)$



ACTIVITY 2 - OUTCOMES 4 and 5

Draw a box on the board.

I have a box that can do strange things to numbers. I will put a number in the top hole, turn the crank, and a number comes out the side. I am going to drop in a 4. Turn the crank. Look what came out! A 6.

What did the box do to the number 4 when I turned the crank?

Added 2:

$$4 + 2 = 6$$

Multiplied by $1\frac{1}{2}$:

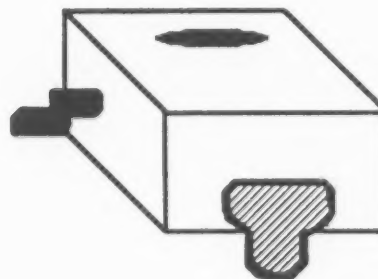
$$4 \times \frac{3}{2} = 6$$

Multiplied by 2 and subtracted 2:

$$(2 \times 4) - 2 = 6$$

What is this box doing to the numbers?

Put in another number and try it. Put in a 7 and turn the crank. A 9 comes out.



ACTIVITY 2 - OUTCOMES 4 and 5 (Cont'd)

What did the box do to the number 7?

Added 2:

$$7 + 2 = 9$$

Multiplied by 2 and subtracted 5:

$$(2 \times 7) - 5 = 9$$

But what does the box do if it has to perform the same operations every time the crank is turned? It added 2 to the number put in, and it will continue to do so. This could be written as: $f(x) = x + 2$.

The box may be called a function box. Remember: it will sit here all day and perform, but it will never do anything other than add 2 to the number put in it.

Build another function box and see what it will do.



This is a modern version of a function box. It is a push button, compact, faster model.

Drop in a 5 and punch the buttons. The number 13 comes out.

What could the box do to a 5 to get a 13?

Add 8:

$$5 + 8 = 13$$

Multiply by 3 and subtract 2:

$$(3 \times 5) - 2 = 13$$

Multiply by 2 and add 3:

$$(2 \times 5) + 3 = 13$$

Now put in a -3 and punch the buttons. The number -3, comes out.

What did the function box do to -3?

It couldn't add 8, because $-3 + 8 = 5$.

It couldn't multiply by 3 and subtract 2, because $3(-3) - 2 = -11$.

It could multiply by 2 and add 3.

$$2(-3) + 3 =$$

$$-6 + 3 =$$

$$-3$$

ACTIVITY 2 - OUTCOMES 4 and 5 (Cont'd)

Write the algebraic equation for this function:

$$f(x) = 2x + 3$$

The pairing of the numbers is called a "relation." In the first box we put in 4 and 6 came out, and then we put in 7 and 9 came out.

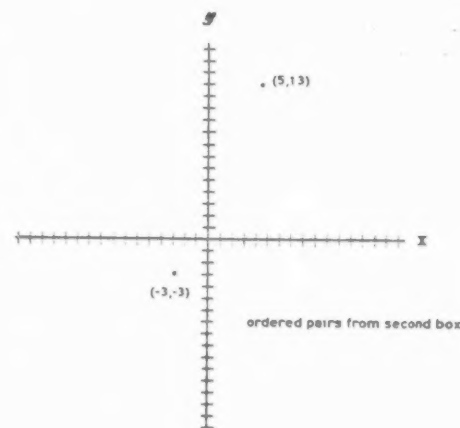
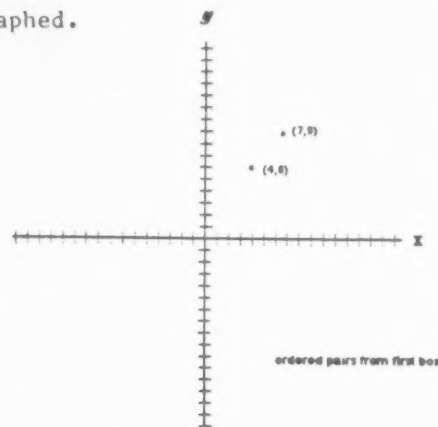
This may be written algebraically as a set of ordered pairs: $\{(4,6); (7,9)\}$ or

4	6
7	9

What would be the set of ordered pairs for the second function box? $\{(5,13); (-3, -3)\}$ or

5	13
-3	-3

These relations can be graphed.



Look at this set of ordered pairs. $\{(3,9); (4,12); (3,-5)\}$ Is there anything strange about the pairs of numbers? There must have been a short circuit in the function box. A 3 was put in one time and came out a nine. The next time a three was put in, it came out a -5. This cannot happen. Someone has tinkered with the function box. It is no longer performing the same function. This relation is not a function.

ACTIVITY 2 - OUTCOMES 4 and 5 (Cont'd)

Example 1: Study the given ordered pairs and determine what the function would be. Write the function algebraically.

0	0
-5	25
$\frac{1}{4}$	$\frac{1}{16}$
1	1

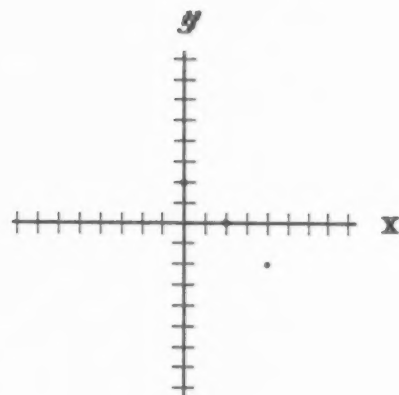
$$f(x) = x^2$$

0	-2
-2	2
4	14
8	62

$$f(x) = x^2 - 2$$

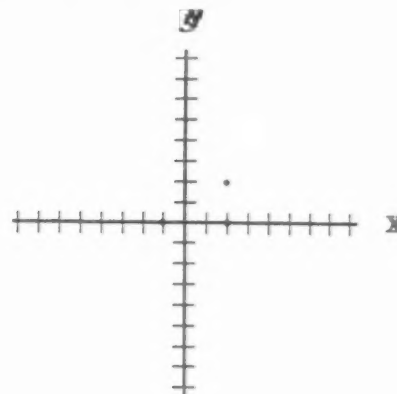
Example 2: Look at the graph of each set of ordered pairs and state if the relation is a function.

(a)



Yes

(b)



No! $f(2) = 0$ and 2
Short circuit!

ALGEBRA I

AREA: FIRST DEGREE EQUATIONS AND INEQUALITIES CONTAINING TWO VARIABLES

- OUTCOMES: 6. Graph linear equations in two variables.
7. Graph linear inequalities in two variables.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Know how to plot points in the coordinate plane.

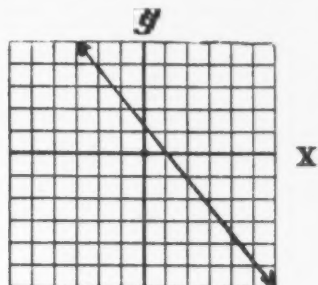
Steps to Success

1. The students will develop an understanding that every ordered pair on a straight line will satisfy the equation for that line.
2. The students will graph linear equations by plotting points of ordered pairs that satisfy the given equation and by graphing linear equations using the slope-intercept form.
3. The students will use the same procedure for graphing linear inequalities but will show the inequality by broken or solid lines and shading.

ACTIVITY 1 - OUTCOME 6

TYPE 1: GRAPHING A LINEAR EQUATION BY PLOTTING POINTS

Look closely at the straight line on the coordinate plane below.



NOTE: If the straight line is extended in both directions, it will eventually intersect every value of x and every value of y .

There would be two special cases:

- (1) When the straight line is parallel to the x -axis, it would only have one value for y .
- (2) When the straight line is parallel to the y -axis, it would only have one value for x .

Look at the straight line again. If the value of any one coordinate is known, would those values for x and y satisfy the equation for that straight line? Yes.

Example 1: Does the point $(3,3)$ lie on the line whose equation is $2x - y = 3$?

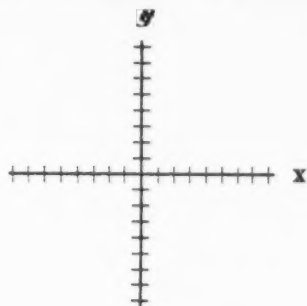
$$\begin{aligned} 2x - y &= 3 \\ 2(3) - 3 &= 3 \\ 6 - 3 &= 3 \\ 3 &= 3 \end{aligned}$$

Yes, it does lie on the line.

ACTIVITY 1 - OUTCOME 6 (Cont'd)

Have you ever hunted a needle in a haystack? Let us consider the coordinate plane to be a haystack and a straight line to be the needle.

If we are given an equation for a straight line, such as $x + y = 4$, and told to graph the equation, how do we know where that line exists on the coordinate plane?



It is somewhere on that coordinate plane. Pick three values of x since the line will eventually intersect every value of x .

x	y
-1	
0	
2	

Now substitute each value chosen for x into the equation and solve for the value of y .

When $x = -1$
 $x + y = 4$
 $-1 + y = 4$
 $y = 5$
 $(-1, 5)$

When $x = 2$
 $x + y = 4$
 $0 + y = 4$
 $y = 4$
 $(0, 4)$

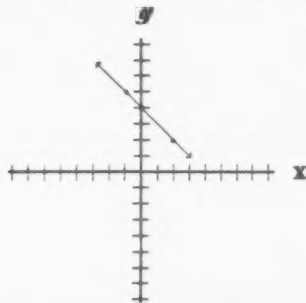
When $x = 0$
 $x + y = 4$
 $2 + y = 4$
 $y = 2$
 $(2, 2)$

Now there is a chart for the coordinates of the line whose equation is $x + y = 4$.

x	y
-1	5
0	4
2	2

ACTIVITY 1 - OUTCOME 6 (Cont'd)

Having found three coordinates that lie on the line, now plot those points and draw a straight line through them.



Thus, we have found our needle in a haystack.

Example 2: Graph the equation $4x + 2y = 8$.

Substitute in points for x , and solve the equation for y .

$$\begin{aligned} 4x + 2y &= 8 \\ 2y &= -4x + 8 \\ y &= -2x + 4 \end{aligned}$$

Choose three points for x .

x	y
-1	
0	
2	

$$\begin{aligned} x &= 1 \\ y &= -2x + 4 \\ y &= -2(-1) + 4 \\ y &= +2 + 4 \\ y &= 6 \\ (-1, 6) \end{aligned}$$

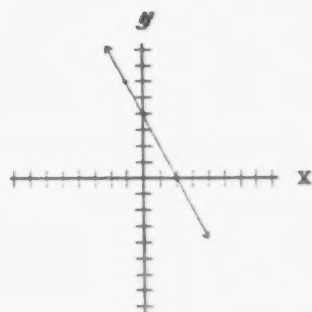
$$\begin{aligned} x &= 0 \\ y &= -2x + 4 \\ y &= -2(0) + 4 \\ y &= 4 \\ (0, 4) \end{aligned}$$

$$\begin{aligned} x &= 2 \\ y &= -2x + 4 \\ y &= -2(2) + 4 \\ y &= -4 + 4 \\ y &= 0 \\ (2, 0) \end{aligned}$$

x	y
-1	6
0	4
2	0

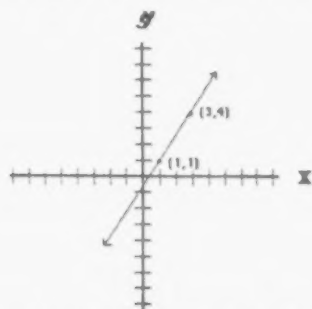
ACTIVITY 1 - OUTCOME 6 (Cont'd)

Plot these points on the coordinate plane.



TYPE 2: GRAPHING LINEAR EQUATIONS USING THE SLOPE-INTERCEPT FORM

The slope of a line is the ratio of the $\frac{\text{vertical change}}{\text{horizontal change}}$ = $\frac{\text{change in } y}{\text{change in } x}$ = $\frac{y_2 - y_1}{x_2 - x_1} = m$.

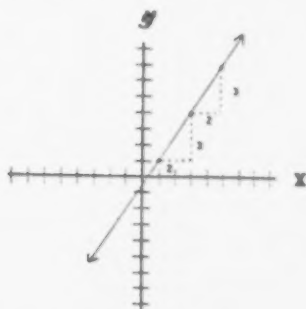


$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{4 - 1}{3 - 1}$$

$$m = \frac{3}{2}$$

This means that in the graph from one point to another, the value of y changes 3 and the value of x changes 2.



ACTIVITY 1 - OUTCOME 6 (Cont'd)

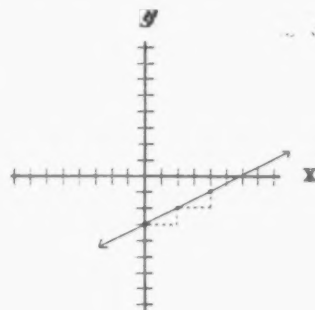
Knowing how to graph the slope helps in drawing the graph of a line when given only one point and the slope.

For example, if we know where the line crosses the y axis, we can start there and plot the points. If the line crosses the y axis at -3 and the slope is $1/2$, can we graph the line? Yes.

First plot the point where the line crosses the y axis = -3.

Then plot the slope of $1/2 = \frac{\text{change in } y}{\text{change in } x}$ from that point.

Finally draw the line through the points.



The point where the line crosses the y axis is called the y-intercept. What is the value of x any time a line crosses the y axis? Zero

One way to write an equation for a straight line is $y = mx + b$ where m = slope and b = y-intercept. This is called the slope-intercept form.

It is a simple process to take any linear equation and place in the form of $y = mx + b$ by solving the equation for y.

$$\begin{aligned} -3x + 4y &= 12 \\ 4y &= 3x + 12 \\ y &= \frac{3}{4}x + 3 \end{aligned}$$

where $m = 3/4$ and $b = 3$

ACTIVITY 1 - OUTCOME 6 (Cont'd)

Example: Graph the equation $6x + 2y = -4$ by using the slope-intercept form.

$$6x + 2y = -4$$

$$2y = -6x - 4$$

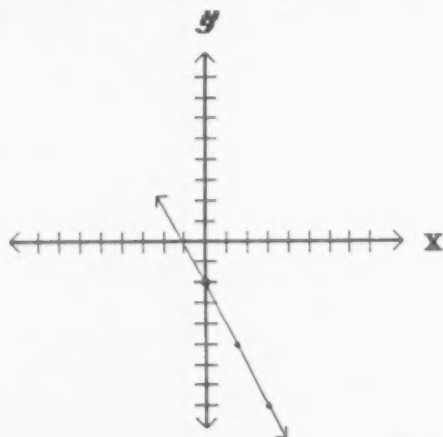
$$y = -3x - 2$$

$$m = -3/1 \text{ and } b = -2$$

Plot y-intercept = -2

Plot the slope = -3/1.

Draw the line.



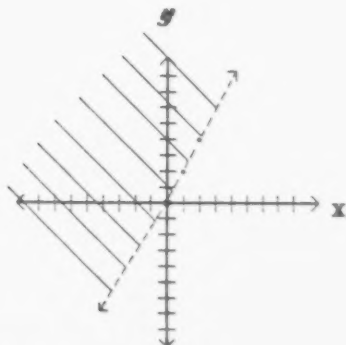
ACTIVITY 2 - OUTCOME 7

Linear inequalities are graphed like linear equalities. The only exceptions are as follows: the line is graphed by using a broken line, not a solid line; and shading is used to indicate less than or greater than.

Example 1: Graph $y > 2x$

$$m = 2$$

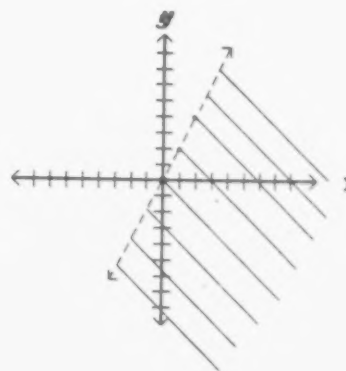
$$b = 0$$



Example 2: Graph $y < 2x$

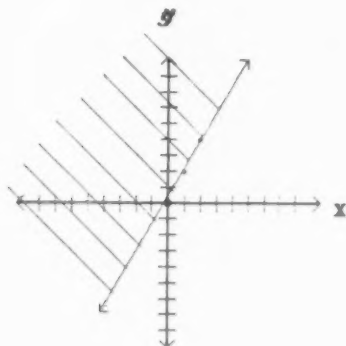
$$m = 2$$

$$b = 0$$

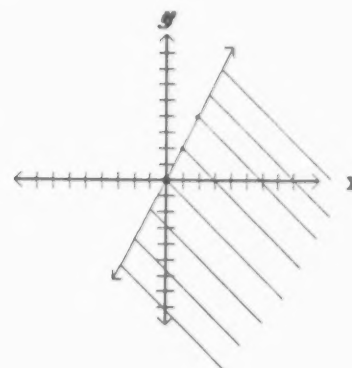


ACTIVITY 2 - OUTCOME 7 (Cont'd)

Example 3: Graph $y \geq 2x$
 $m = 2$
 $b = 0$



Example 4: Graph $y \leq 2x$
 $m = 2$
 $b = 0$



NOTE: If ever in doubt about which side of the line to shade in an inequality, pick one point on either side of the line and substitute that point into the inequality to see if it satisfies the inequality. If it does satisfy the inequality, then shade that side.

ALGEBRA I

AREA: FIRST DEGREE EQUATIONS AND INEQUALITIES CONTAINING TWO VARIABLES

- OUTCOMES:
8. Use graphs to solve systems of linear equations in two variables.
 9. Graph the solution set of a system of two linear inequalities in two variables.
 10. Use the substitution method to solve systems of linear equations in two variables.
 11. Use the addition or subtraction method to solve systems of linear equations in two variables.
 12. Use multiplication with the addition or subtraction method to solve systems of linear equations.
 13. Use systems of linear equations in two variables to solve problems.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Be able to graph linear equations and linear inequalities.
2. Solve linear equations.

Steps to Success

1. The students' abilities to transfer their knowledge for solving linear equations to solving two equations with two unknowns will ensure success in solving simultaneous linear equations and linear inequalities.
2. Two equations with two unknowns will be solved by graphing, substitution, addition or subtraction, and multiplication with addition or subtraction methods.
3. The students will use their knowledge for solving word problems and solving equations simultaneously to solve additional word problems with two equations.

ACTIVITY 1 - OUTCOMES 8, 9, 10, 11, and 12

Until this point in algebra, linear equations could be solved by performing the inverse operations. Also quadratic equations could be solved by factoring. There is still another type of equations to solve. They are linear equations with two variables.

ACTIVITY 1 - OUTCOMES 8, 9, 10, 11, and 12 (Cont'd)

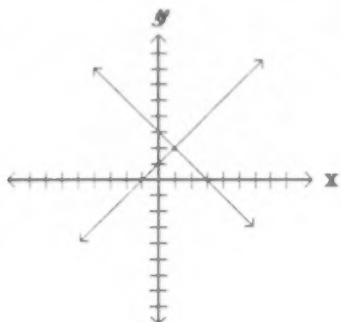
$$\begin{aligned}x + y &= 10 \\ 2x - y &= 8\end{aligned}$$

Because two equations impose two conditions on the variables, they are called simultaneous equations. For example, the value of x plus the value of y must equal ten while the same value of y subtracted from twice the value of x is equal to 8. It is necessary to solve for both variables in the system of equations. There are at least four ways to solve equations simultaneously.

TYPE 1: SOLVING SYSTEMS OF EQUATIONS BY GRAPHING

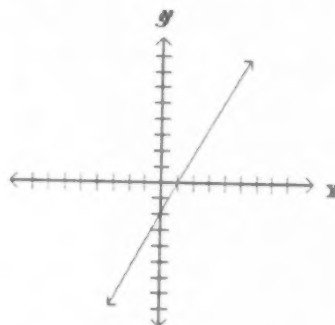
When two linear equations are graphed on one coordinate plane, there are three different cases or solutions that might occur.

Case 1:



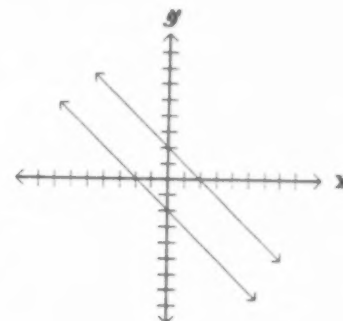
The graphs of the linear equations may intersect at one point. If they do, then that point is their solution. It will satisfy both equations.

Case 2:



The graphs of the linear equations may coincide. This means there are many solutions or an infinite number of points which would satisfy both equations.

Case 3:



The graphs of the linear equations may be parallel. They have no solution in common.

ACTIVITY 1 - OUTCOMES 8, 9, 10, 11, and 12 (Cont'd)

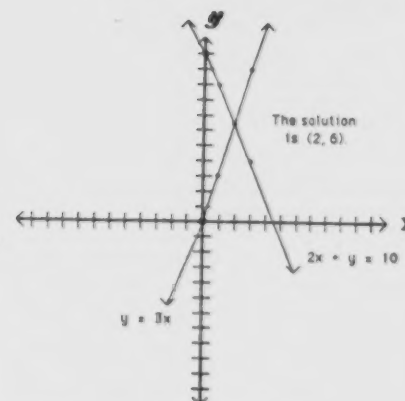
Example 1: Solve the system of equations by graphing.

$$\begin{aligned} y &= 3x \\ 2x + y &= 10 \end{aligned}$$

Step 1: Solve each equation for y and find m and b.

$$\begin{array}{ll} y = 3x & 2x + y = 10 \\ m = 3 & y = -2x + 10 \\ b = 0 & m = -2 \\ & b = 10 \end{array}$$

Step 2: Graph each equation using the intercept and the slope.



The solution may be checked. Substitute the values of x and y into the original equations to determine if they satisfy both equations.

(2,6)

$$\begin{array}{ll} y = 3x & 2x + y = 10 \\ 6 = 3(2) & 2(2) + 6 = 10 \\ 6 = 6 & 10 = 10 \end{array}$$

Example 2: Solve the system of linear inequalities by graphing.

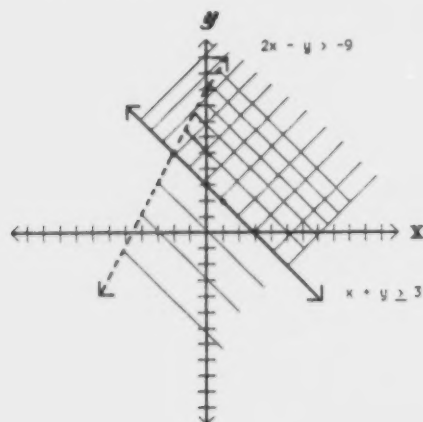
$$\begin{aligned} x + y &\geq 3 \\ 2x - y &> -9 \end{aligned}$$

ACTIVITY 1 - OUTCOMES 8, 9, 10, 11, and 12 (Cont'd)

Step 1: Solve each inequality for y.

$$\begin{array}{l} x + y \geq 3 \\ y \geq -x + 3 \\ m = -1 \\ b = 3 \end{array} \qquad \begin{array}{l} 2x - y > -9 \\ -y > -2x - 9 \\ y < 2x + 9 \\ m = 2 \\ b = 9 \end{array}$$

Step 2: Graph each inequality.



The solution is the part of the graph which is common to both inequalities.

TYPE 2: SOLVING SYSTEMS OF EQUALITIES BY SUBSTITUTION

Use the substitution method when one of the equations is easily solved in terms of one variable making it easy to substitute into the other equation.

ACTIVITY 1 - OUTCOMES 8, 9, 10, 11, and 12 (Cont'd)

Example 3: $a = b - 2$
 $3b - a = 16$

Step 1: Solve the first equation for a.

$$a = b - 2$$

Step 2: Substitute the value of a, which is $(b - 2)$, into the second equation for the variable a.

$$3b - a = 16$$
$$3b - (b - 2) = 16$$

Step 3: Solve the resulting equation for b.

$$3b - b + 2 = 16$$
$$2b = 14$$
$$b = 7$$

Step 4: Take this value of b and substitute it into one of the original equations and solve for a.

$$a = b - 2$$
$$a = (7) - 2$$
$$a = 5$$

The solutions to the equations are: $a = 5$ and $b = 7$.

To check, substitute both values in both equations.

$a = b - 2$	$3b - a = 16$
$5 = 7 - 2$	$3(7) - 5 = 16$
$5 = 5$	$21 - 5 = 16$
	$16 = 16$

ACTIVITY 1 - OUTCOMES 8, 9, 10, 11, and 12 (Cont'd)

TYPE 3: SOLVING SYSTEMS OF EQUATIONS BY ADDITION OR SUBTRACTION

Use the addition or subtraction method when one variable will cancel if the two equations are added or subtracted.

Example: $a + b = 25$
 $a - b = -5$

NOTE: When the like term of the two equations are added, the variable b will cancel, leaving a linear equation with one variable.

$$\begin{array}{r} a + b = 25 \\ a - b = -5 \\ \hline 2a = 20 \end{array}$$

Solve the resulting equation.

$$\begin{array}{l} 2a = 20 \\ a = 10 \end{array}$$

Since the value of a is known, substitute its value into one of the original equations and solve for b .

$$\begin{array}{l} a + b = 25 \\ 10 + b = 25 \\ b = 15 \end{array}$$

To check, substitute the value for a and b into both equations.

$$\begin{array}{lcl} a + b = 25 & a = 10; b = 15 & a - b = -5 \\ 10 + 15 = 25 & & 10 - (15) = -5 \\ 25 = 25 & & -5 = -5 \end{array}$$

ACTIVITY 1 - OUTCOMES 8, 9, 10, 11, and 12 (Cont'd)

TYPE 4: SOLVING SYSTEMS OF EQUATIONS BY MULTIPLICATION WITH ADDITION OR SUBTRACTION

Sometimes two equations do not lend themselves to adding or subtracting to determine their solution. It may be evident that multiplying one or both equations by a constant, will make the coefficients of a variable equal, so that addition or subtraction may then be used to cancel one of the variables.

$$\begin{aligned} 3m + 2n &= 24 \\ 2m - 3n &= -1 \end{aligned}$$

NOTE: Neither the m nor n will cancel simply by adding or subtracting the two equations. If the top equation is multiplied by 3 and the bottom equation by 2, then addition is used and the n in the two equations will cancel.

NOTE: Don't forget to multiply every term on both sides of the equation.

$$\begin{array}{ll} \text{Multiply by 3:} & 3m + 2n = 24 \\ \text{Multiply by 2:} & 2m - 3n = -1 \\ \hline & \\ \text{Solve for m:} & 9m + 6n = 72 \\ & 4m - 6n = -2 \\ \hline 13m & = 70 \\ m & = 70/13 \end{array}$$

To solve for n , substitute the value of m into one of the original equations.

$$\begin{aligned} 3m + 2n &= 24 \\ 3\left(\frac{70}{13}\right) + 2n &= 24 \\ \frac{210}{13} + 2n &= 24 \\ 2n &= 24 - \frac{210}{13} \\ 2n &= \frac{312}{13} - \frac{210}{13} \\ 2n &= \frac{102}{13} \\ \frac{1}{2}(2n) &= \frac{1}{2}\left(\frac{102}{13}\right) \\ n &= \frac{51}{13} \end{aligned}$$

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ACTIVITY 1 - OUTCOMES 8, 9, 10, 11, and 12 (Cont'd)

$$\text{Check: } m = \frac{70}{13}, \quad n = \frac{51}{13}$$

$$\begin{aligned} 3m + 2n &= 24 \\ 3\left(\frac{70}{13}\right) + 2\left(\frac{51}{13}\right) &= 24 \\ \frac{210}{13} + \frac{102}{13} &= 24 \\ \frac{312}{13} &= 24 \\ 24 &= 24 \end{aligned}$$

$$\begin{aligned} 2m - 3n &= -1 \\ 2\left(\frac{70}{13}\right) - 3\left(\frac{51}{13}\right) &= -1 \\ \frac{140}{13} - \frac{153}{13} &= -1 \\ \frac{-13}{13} &= -1 \\ -1 &= -1 \end{aligned}$$

ACTIVITY 2 - OUTCOME 13

Reading problems which have two unknowns can now be solved. They will be set up similarly to the reading problems with one variable. One of the methods just discussed will be used to solve the equations simultaneously.

Example 1: John spent two more hours in school than he did working on his delivery route. He spent a total of 12 hours doing both. How many hours did he spend doing each?

Step 1: Represent the unknowns:

x = number of hours in school
 y = number of hours on delivery route

Step 2: Write the equations:

$$\begin{aligned} x + y &= 12 && \{\text{time spent doing both}\} \\ x - y &= 2 && \{\text{the difference in time of the two}\} \end{aligned}$$

Step 3: Choose the method for solving: addition

$$\begin{array}{rcl} x + y & = & 12 \\ x - y & = & 2 \\ \hline 2x & = & 14 \\ x & = & 7 \end{array} \qquad \begin{array}{rcl} x + y & = & 12 \\ 7 + y & = & 12 \\ y & = & 5 \end{array}$$

Step 4: Solve:

$$x = 7$$

Step 5: Check:

$$\begin{array}{rcl} x + y & = & 12 \\ 7 + 5 & = & 12 \\ 12 & = & 12 \end{array} \qquad \begin{array}{rcl} x - y & = & 2 \\ 7 - 5 & = & 2 \\ 2 & = & 2 \end{array}$$

Thus John went to school for 7 hours and worked for 5 hours.

ACTIVITY 2 - OUTCOME 13 (Cont'd)

Example 2: The sum of two numbers is 80. If twice the larger number is subtracted from 4 times the smaller number, the result is 8. Find the two numbers.

Step 1: Represent the unknowns:

x = the smaller number

y = the larger number

Step 2: Write the equations:

$$x + y = 80 \text{ {Their sum is 80}}$$

$$4x - 2y = 8 \text{ {Twice the larger is subtracted from 4 times the smaller.}}$$

Step 3: Choose the method for solving: multiplication with addition.

$$2x + 2y = 160$$

$$4x - 2y = 8$$

Step 4: Solve:

$$6x = 168$$

$$x = 28$$

$$x + y = 80$$

$$28 + y = 80$$

$$y = 52$$

Step 5: Check:

$$x + y = 80$$

$$28 + 52 = 80$$

$$80 = 80$$

$$4x - 2y = 8$$

$$4(28) - 2(52) = 8$$

$$112 - 104 = 8$$

$$8 = 8$$

Thus the smaller number is 28 and the larger number is 52.

ALGEBRA I

AREA: REAL NUMBERS

- OUTCOMES:
1. Define real numbers.
 2. Give examples of real numbers.
 3. Determine whether a set of given numbers is a subset of the real numbers.
 4. Define irrational numbers.
 5. Determine whether a set of given numbers is a subset of the irrational numbers.
 6. Solve simple radical equations.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Have a general knowledge of sets of numbers.
2. Know the postulates and properties of real numbers.
3. Be familiar with terminating, non-terminating, repeating, and non-repeating decimals.

Steps to Success

1. The real numbers are the set of all positive numbers, negative numbers, and zero.
2. Rational numbers are real numbers that can be expressed as a/b where $b \neq 0$ and where a and b are integers.
3. Irrational numbers are real numbers which cannot be expressed as a/b , where $b \neq 0$ and where a and b are integers.
4. Every positive number has two square roots: one positive and the other negative. The positive square root of a number is called the principal square root.
5. A radical is in simplest form when:
 - a. no integral radicand has a square factor other than 1;
 - b. no fractions are under the radical sign; and
 - c. no radicals are in a denominator of a fraction.
6. Like radicals have the same root index and the same radicand.
7. To solve radical equations: (1) isolate the radical; (2) square both members; (3) solve the resulting equation; and (4) check the solution.

ACTIVITY 1 - OUTCOMES 1, 2, 3, 4, and 5

Draw a large barrel on the board.



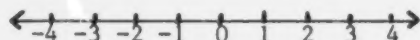
Tell the students that this barrel contains every number that exists.

Ask the students to list sets of numbers that have been discussed and used in algebra.

Examples: counting numbers, fractions, decimals, positive numbers, negative numbers, etc.

There is a large set of numbers that should be examined. They are called the real numbers.

Remember the properties studied earlier and used ever since that time? These properties represent numbers that are members of the set of real numbers. The real numbers correspond to a unique point on the number line. Can a definition of real numbers be made by looking at the number line?



All positive numbers, negative numbers, and zero.

Many, years ago when man started to experiment with numbers, he soon learned to add, subtract, and multiply them. Man also began to divide numbers, but he found that in division all numbers did not come out even. He represented these numbers as fractions. Soon he found the need to go further than fractional representation. He called these numbers decimal numbers. He found three situations occurring: (1) the decimal terminated, (2) the decimal repeated, or (3) the decimal neither terminated nor repeated.

Man needed to give these numbers a name. He called the easy set of numbers the rational numbers. He called the more difficult set of numbers irrational numbers.

ACTIVITY 1 - OUTCOMES 1, 2, 3, 4, and 5 (Cont'd)

Rational Numbers	Irrational Numbers
Positive integers	Positive numbers
Negative integers	Negative numbers
Zero	
Numbers that could be expressed as a/b where a and b are integers and $b \neq 0$	Numbers that could not be expressed as a/b where a and b are integers and $b \neq 0$
Numbers that could be plotted on the number line	Numbers that could be associated with the number line but are approximated
Terminating decimal numbers	Decimal numbers that neither terminate nor repeat
Repeating decimal numbers	
Numbers such as $\sqrt{n}$ when n is a perfect square	Numbers such as $\sqrt{n}$ when n is not a perfect square

State whether the number is rational or irrational:

Example 1: .59	(Rational)	Example 2: .575775777...	(Irrational)
Example 3: $1/3$	(Rational)	Example 4: .21222324	(Irrational)
Example 5: .35	(Rational)	Example 6: $\sqrt{7}$	(Irrational)
Example 7: $\sqrt{25}$	(Rational)		

ACTIVITY 2 - OUTCOME 6

Use the product and quotient properties of square roots with the commutative and associative properties to multiply, divide, and simplify radicals.

The product property of radicals is: $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

Example 1: $\sqrt{144} = 12$

Example 2: $\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$

Example 3: $\sqrt{3} \cdot \sqrt{7} = \sqrt{21}$

Example 4: $\sqrt{2} \cdot \sqrt{18} = \sqrt{36} = 6$

The quotient property of radicals is: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Example 5: $\sqrt{\frac{49}{81}} = \frac{\sqrt{49}}{\sqrt{81}} = \frac{7}{9}$

Example 6: $\frac{\sqrt{28}}{\sqrt{7}} = \sqrt{\frac{28}{7}} = \sqrt{4} = 2$

NOTE: Sometimes a radical may be in the denominator of a fraction that cannot be simplified. A process called rationalizing the denominator to simplify the fraction should be used.

Example 7: $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{9}} = \frac{\sqrt{3}}{3}$

Like radicals can be combined by adding or subtracting their coefficients and multiplying that coefficient by the common radical.

Example 8: $8\sqrt{13} + 3\sqrt{13} = 11\sqrt{13}$

Example 9: $2\sqrt{2} + \sqrt{32} =$

$2\sqrt{2} + \sqrt{16 \cdot 2} =$

$2\sqrt{2} + 4\sqrt{2} =$

300

$6\sqrt{2}$

301

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ACTIVITY 2 - OUTCOME 6 (Cont'd)

$$\begin{aligned}\text{Example 10: } 3\sqrt{8} + \sqrt{50} - \sqrt{3} &= \\ 3\sqrt{4 \cdot 2} + \sqrt{25 \cdot 2} - \sqrt{3} &= \\ 3(2)\sqrt{2} + 5\sqrt{2} - \sqrt{3} &= \\ 6\sqrt{2} + 5\sqrt{2} - \sqrt{3} &= \\ 11\sqrt{2} - \sqrt{3} &= \end{aligned}$$

Find the product of binomials containing radicals.

$$\begin{aligned}\text{Example 11: } (5 + \sqrt{2})(5 - \sqrt{2}) &= \\ 25 - 5\sqrt{2} + 5\sqrt{2} - \sqrt{4} &= \\ 25 - \sqrt{4} &= \\ 25 - 2 &= \\ 23 &= \end{aligned}$$

$$\begin{aligned}\text{Example 12: } 2\sqrt{5}(2\sqrt{10} + \sqrt{5}) &= \\ (2)(2)\sqrt{5 \cdot 10} + 2\sqrt{5 \cdot 5} &= \\ 4\sqrt{50} + 2\sqrt{25} &= \\ 4\sqrt{25 \cdot 2} + 2(5) &= \\ 4(5)\sqrt{2} + 10 &= \\ 20\sqrt{2} + 10 &= \end{aligned}$$

ACTIVITY 2 - OUTCOME 6 (Cont'd)

When the denominator of a fraction is a binomial containing a radical, the denominator can be rationalized by multiplying the numerator and denominator by its binomial opposite, its conjugate.

Example 13:

$$\begin{aligned} \frac{1}{1-\sqrt{5}} & \cdot \frac{1+\sqrt{5}}{1+\sqrt{5}} = \frac{1+\sqrt{5}}{(1-\sqrt{5})(1+\sqrt{5})} = \frac{1+\sqrt{5}}{1-\sqrt{25}} \\ & = \frac{1+\sqrt{5}}{1-5} \\ & = \frac{1+\sqrt{5}}{-4} \end{aligned}$$

Example 14: $4\sqrt{x} = 36$

Step 1: Isolate the radical:

$$\begin{aligned} 4\sqrt{x} &= 36 \\ x &= 9 \end{aligned}$$

Step 2: Square both sides:

$$(\sqrt{x})^2 = (9)^2$$

Step 3: Solve the resulting equation:

$$\begin{aligned} \sqrt{x^2} &= 81 \\ x &= 81 \end{aligned}$$

To check:

$$\begin{aligned} 4\sqrt{x} &= 36 \\ 4\sqrt{81} &= 36 \\ 4(9) &= 36 \\ 36 &= 36 \end{aligned}$$

ACTIVITY 2 - OUTCOME 6 (Cont'd)

Example 15: Solve each equation

$$\begin{aligned}(1) \quad \sqrt{a+2} &= 4 \\ (\sqrt{a+2})^2 &= (4)^2 \\ a+2 &= 16 \\ a &= 14\end{aligned}$$

$$\begin{aligned}\text{Check: } \sqrt{a+2} &= 4 \\ \sqrt{14+2} &= 4 \\ \sqrt{16} &= 4 \\ 4 &= 4\end{aligned}$$

$$\begin{aligned}(2) \quad \sqrt{2b} - 1 &= -3 \\ \sqrt{2b} &= -2 \\ (\sqrt{2b})^2 &= (-2)^2 \\ \sqrt{4b^2} &= 4 \\ 2b &= 4 \\ b &= 2\end{aligned}$$

$$\begin{aligned}\text{Check: } \sqrt{2b} - 1 &= -3 \\ \sqrt{2 \cdot 2} - 1 &= -3 \\ \sqrt{4} - 1 &= -3 \\ 2 - 1 &= -3 \\ 1 &= -3\end{aligned}$$

Not 2 is not the solution.

$$\begin{aligned}(3) \quad 3\sqrt{5c^2 - 11} &= 9 \\ \sqrt{5c^2 - 11} &= 3 \\ (\sqrt{5c^2 - 11})^2 &= (3)^2 \\ 5c^2 - 11 &= 9\end{aligned}$$

$$\begin{aligned}5c^2 &= 20 \\ c^2 &= 4 \\ c &= \pm\sqrt{4} \\ c &= \pm 2\end{aligned}$$

$$\begin{aligned}\text{Check: } 3\sqrt{5(2)^2 - 11} &= 9 \\ 3\sqrt{20 - 11} &= 9 \\ 3\sqrt{9} &= 9 \\ 3 \cdot 3 &= 9 \\ 9 &= 9\end{aligned}$$

ALGEBRA I

AREA: QUADRATIC EQUATIONS

- OUTCOMES:
1. Transform quadratic equations into standard form: $ax^2 + bx + c = 0$.
 2. Solve quadratic equations by factoring.
 3. Solve quadratic equations by completing the square.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Recognize a quadratic equation.
2. Have a good understanding of powers and roots.

Steps to Success

1. Use the following steps to solve the quadratic equation by completing the square:
 - a. Write the equation in the standard form of $ax^2 + bx + c = 0$.
 - b. Eliminate the coefficient of the quadratic term if there is one.
 - c. Eliminate the constant term from the left member.
 - d. Take one-half the linear term's coefficient, square it, and add it to both members of the equation.
 - e. Write the left member in factored form (square of a binomial). Simplify the right member.
 - f. Use the property of square roots to determine the solutions.
 - g. Simplify and check the solutions.

ACTIVITY I - OUTCOMES 1, 2, and 3

When in its simplest form, an equation whose unknown variable has a highest degree of two is called a quadratic equation. In its simplest form it will look like $ax^2 + bx + c = 0$.

Where ax^2 is the quadratic term, bx is the linear term and c is the constant term.

ACTIVITY I - OUTCOMES 1, 2, and 3 (Cont'd)

When one of the terms is missing from the quadratic equation, it is called an incomplete quadratic equation. Example: $ax^2 + bx = 0$

$$\text{or} \\ ax^2 + c = 0$$

Previously solved quadratic equations look like these:

$$\begin{aligned} 4x^2 - 16 &= 0 \\ 4x^2 &= 16 \\ x^2 &= 4 \\ x &= \pm\sqrt{4} \\ x &= \pm 2 \end{aligned}$$

Complete quadratic equations of the form $ax^2 + bx + c = 0$ have been solved by factoring.

$$\begin{aligned} a^2 + 6a + 5 &= 0 \\ (a + 5)(a + 1) &= 0 \\ a + 5 = 0 \quad a + 1 &= 0 \\ a = -5 \quad a &= -1 \end{aligned}$$

Some equations may not lend themselves to factoring as easily as others. When this happens, a way to solve the quadratic equation is called completing a trinomial square.

The point in solving a quadratic equation is to derive a solution.

When the quadratic equation is not in perfect form, transform it into that form and solve the equation.

Solve: $2x^2 + 3x - 2 = 0$

Step 1. Transform the equation
into standard form of:

$$ax^2 + bx + c = 0$$

Step 2. Eliminate the coefficient
of the quadratic term:

$$\frac{2x^2}{2} + \frac{3x}{2} - \frac{2}{2} = \frac{0}{2}$$

$$x^2 + \frac{3x}{2} - 1 = 0$$

ACTIVITY I - OUTCOMES 1, 2, and 3 (Cont'd)

- Step 3. Eliminate the constant term from the left number:

$$x^2 + \frac{3x}{2} = 1$$

- Step 4. Take one-half the linear term coefficient, square it, and add it to both sides:

$$x^2 + \frac{3x}{2} + \frac{9}{16} = \frac{9}{16} + 1$$

$$\begin{aligned} 1/2(3/2) &= 3/4 \\ (3/4)^2 &= 9/16 \end{aligned}$$

NOTE: The left member is now a trinomial square.

- Step 5. Write the left member in factored form and simplify the right member:

$$\left(x + \frac{3}{4}\right)^2 = \frac{25}{16}$$

- Step 6. Use the property of square root:

$$x + \frac{3}{4} = \pm \sqrt{\frac{25}{16}}$$

- Step 7. Simplify and check:

$$\begin{aligned} x &= -\frac{3}{4} \pm \frac{5}{4} \\ x &= +\frac{1}{2} \text{ or } -2 \end{aligned}$$

$$2x^2 + 3x - 2 = 0$$

$$\text{if } x = \frac{1}{2}$$

$$2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) - 2 = 0$$

$$\text{if } x = -2$$

$$2(-2)^2 + 3(-2) - 2 = 0$$

$$2\left(\frac{1}{4}\right) + \frac{3}{2} - 2 = 0$$

$$2(4) - 6 - 2 = 0$$

$$\frac{1}{2} + \frac{3}{2} - 2 = 0$$

$$8 - 6 - 2 = 0$$

$$\frac{4}{2} - 2 = 0$$

$$0 = 0$$

$$2 - 2 = 0$$

$$\text{Therefore, } x = \left\{\frac{1}{2}, -2\right\}$$

ACTIVITY I - OUTCOMES 1, 2, and 3 (Cont'd)

Example: Solve by completing a trinomial square.

$$3x^2 - 2x = 2$$

$$3x^2 - 2x - 2 = 0$$

$$x^2 - \frac{2}{3}x - \frac{2}{3} = 0$$

$$x^2 - \frac{2}{3}x = \frac{2}{3}$$

$$x^2 - \frac{2}{3}x + \frac{1}{9} = \frac{2}{3} + \frac{1}{9}$$

$$\left(x - \frac{1}{3}\right)^2 = \frac{7}{9}$$

$$x - \frac{1}{3} = \pm \sqrt{\frac{7}{9}}$$

$$x = \frac{1}{3} \pm \frac{\sqrt{7}}{3}$$

$$x = \frac{1 \pm \sqrt{7}}{3}$$

Both roots are solutions, therefore $x = \left\{ \frac{1 + \sqrt{7}}{3}, \frac{1 - \sqrt{7}}{3} \right\}$.

ALGEBRA I

AREA: QUADRATIC EQUATIONS

- OUTCOMES:
4. Solve quadratic equations by using the quadratic formula.
 5. Graph quadratic functions.
 6. Express word problems as quadratic equations.
 7. Solve word problems which are translated into quadratic equations.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Complete a trinomial square.
2. Plot points on a coordinate plane.
3. Translate word problems into equations.

Steps to Success

1. Take the standard form for a quadratic equation $ax^2 + bx + c = 0$ and, by completing a trinomial square, derive the quadratic formula
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
2. Use the quadratic formula to solve a quadratic equation by substituting the values for a, b, and c and then simplifying the formula.
3. Solve word problems by setting up quadratic equations and solving the equations by factoring, by completing a trinomial square, or by using the quadratic formula.

ACTIVITY I - OUTCOME 4

Where a, b, and c are numerical values, $ax^2 + bx + c = 0$ represents a complete quadratic equation. This is called the standard form of the quadratic equation.

ACTIVITY I - OUTCOME 4 (Cont'd)

Have you ever wondered what would happen if a trinomial square were completed using the standard form $ax^2 + bx + c = 0$?

Step 1. Write the equation in standard form: $ax^2 + bx + c = 0$

Step 2. Eliminate the coefficient of the quadratic term if there is one:

$$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = \frac{0}{a}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Step 3. Eliminate the constant from the left member:

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Step 4. Take one-half the linear term coefficient, square it, add it to both members:

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$\frac{1}{2}\left(\frac{b}{a}\right) = \frac{b}{2a}; \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

Step 5. Write the left member in factored form. Simplify the right member:

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Step 6. Use the property of square roots:

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

Step 7. Simplify and check:

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula is the quadratic formula. It may be used to solve quadratic equations by substituting the value of a, b, and c into the formula and solving for x.

Example: $x^2 - 2x = -5$

Transform given equation into $ax^2 + bx + c = 0$:

Determine the values of a, b, and c:

$$x^2 - 2x - 5 = 0$$

$$a = 1; b = -2; c = -5$$

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ACTIVITY I - OUTCOME 4 (Cont'd)

Substitute the values into the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 + 20}}{2}$$

$$x = \frac{2 \pm \sqrt{24}}{2}$$

$$x = \frac{2 \pm 2\sqrt{6}}{2}$$

$$x = \frac{2(1 \pm \sqrt{6})}{2}$$

$$x = 1 \pm \sqrt{6}$$

Check

$$x^2 - 2x = 5$$

$$\text{If } x = 1 + \sqrt{6}$$

$$\begin{aligned} (1 + \sqrt{6})^2 - 2(1 + \sqrt{6}) &= 5 \\ 1 + 2\sqrt{6} + 6 - 2 - 2\sqrt{6} &= 5 \\ 5 &= 5 \end{aligned}$$

$$\text{If } x = 1 - \sqrt{6}$$

$$\begin{aligned} (1 - \sqrt{6})^2 - 2(1 - \sqrt{6}) &= 5 \\ 1 - 2\sqrt{6} + 6 - 2 + 2\sqrt{6} &= 5 \\ 5 &= 5 \end{aligned}$$

$$\text{Therefore } x = \{1 + \sqrt{6}, 1 - \sqrt{6}\}$$

If the decimal equivalent is needed in the answer, look up the $\sqrt{6}$ and use it to simplify the answer even further.

$$\begin{aligned} 1 + \sqrt{6} \\ 1 + 2.449 \\ 3.449 \end{aligned}$$

$$\begin{aligned} 1 - \sqrt{6} \\ 1 - 2.449 \\ -1.449 \end{aligned}$$

$$x = \{3.449, -1.449\}$$

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ACTIVITY 2 - OUTCOMES 6 and 7

Solving some word problems requires solving quadratic equations. By inspection, determine the most appropriate method for solving the quadratic equation.

Example: The square of a number increased by twice the number equals 8. Find the number.

Represent the unknown: $x = \text{the number}$

Write the equation: $\underbrace{\text{Square of number}}_{x^2} \underbrace{\text{increased by}}_{+} \underbrace{\text{twice the number}}_{2x} \underbrace{\text{equals}}_{=} \underbrace{8}_{8}$

It is a quadratic equation, $x^2 + 2x = 8$
so one of three following
methods may be used to solve
the equation.

ACTIVITY 2 - OUTCOMES 6 and 7 (Cont'd)

Method I Factoring

$$\begin{aligned}x^2 + 2x &= 8 \\x^2 + 2x + 8 &= 0 \\(x + 4)(x - 2) &= 0 \\x + 4 = 0 \text{ or } x - 2 = 0 \\x = -4 \quad \quad x = 2\end{aligned}$$

check $x^2 + 2x = 8$

if $x = 2$

$$\begin{aligned}(2)^2 + 2(2) &= 8 \\4 + 4 &= 8 \\8 &= 8\end{aligned}$$

if $x = -4$

$$\begin{aligned}(-4)^2 + 2(-4) &= 8 \\16 - 8 &= 8 \\8 &= 8\end{aligned}$$

Therefore $x = -4, 2$

Method II Completing a Trinomial Square

$$\begin{aligned}x^2 + 2x &= 8 \\x^2 + 2x + 1 &= 1 + 8 \\(x + 1)^2 &= 9 \\x + 1 &= \pm \sqrt{9} \\x &= -1 \pm 3 \\x = -4 \text{ or } x = 2\end{aligned}$$

Method III Using the Quadratic Formula

$$\begin{aligned}x^2 + 2x - 8 &= 0 \\a = 1 \quad b = 2 \quad c = -8 \\x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\x &= \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-8)}}{2(1)} \\x &= \frac{-2 \pm \sqrt{4 + 32}}{2} \\x &= \frac{-2 \pm \sqrt{36}}{2} \\x &= \frac{-2 \pm 6}{2} \\x &= \frac{-2 + 6}{2} \quad \text{or} \quad \frac{-2 - 6}{2} \\x &= \frac{4}{2} \quad , \quad \frac{-8}{2} \\x &= 2 \quad , \quad -4\end{aligned}$$

An equation may be solved in more than one way. Choose the most appropriate method for the given equation and solve the equation using that method.

ACTIVITY 3 - OUTCOME 5

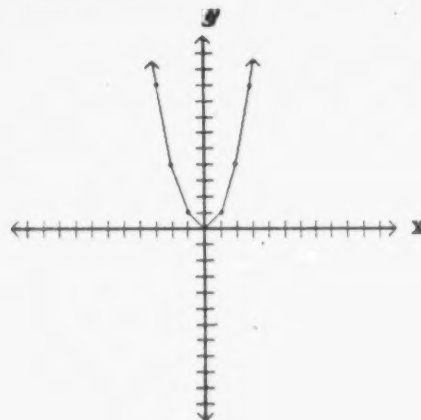
Remember the function box? Here is another function box. Do three things.

- (1) Choose as many different values to put into the box as needed.
- (2) Make a chart to show the values put in and gotten out.
- (3) Plot those points on a coordinate plane.



Example 1: $y = x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

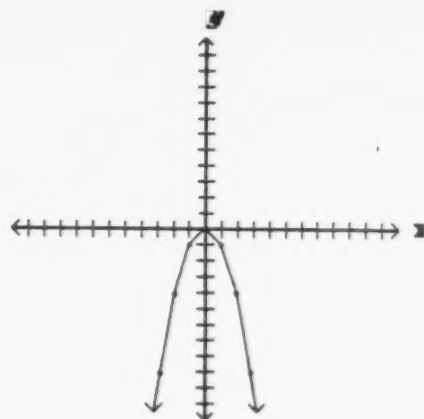


This is called a parabola.

ACTIVITY 2 - OUTCOMES 6 and 7 (Cont'd)

Example 2: Try another function box which performs $y = -x^2$.

x	y
-3	-9
-2	-4
-1	-1
0	0
1	-1
2	-4
3	-9



Ask the students to determine what caused the difference in the graphs and why?

Try several more examples. Have the students determine what will be the change and what caused the change.

Example 3: $y = x^2 + 2$
 $y = x^2 - 2$

Example 4: $y = 2x^2$
 $y = 4x^2$

MATHEMATICS: ALGEBRA I

AREA: Symbols, Numbers, and Numerals

S T A N	C O F F S	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
8	X	Symbols and numerals	1. Use the vocabulary and symbols related to equations, inclusively, as they are introduced.			
	X	Numbers and the number line	2. Graph real numbers on the number line.			
			3. Compare numbers using the number line.			
			4. Relate the absolute value of a number with its distance from the origin on the number line.			
			5. Illustrate the operations of addition and subtraction by the use of arrows representing movement on the number line.			
	X	Order of operations	6. Associate a numeral with an equivalent expression of the same number.			
	X	Use of other mathematical symbols	7. Simplify expressions containing more than one grouping symbol.			
			8. Simplify expressions when grouping symbols are omitted.			

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S T A N	C O F S	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
	X	Use of signed numbers to represent opposite values	1. Given a number, identify its opposite value.			
	X	Absolute value	2. Given a number, state its absolute value.			
	X	Use of signed numbers in fundamental operations	3. Solve problems involving addition, subtraction, multiplication, and division of signed numbers.			
			4. Translate a word problem into mathematical expressions or equations using the correctly signed numbers.			
			5. Solve simple word problems involving signed numbers.			

MATHEMATICS: ALGEBRA I

AREA: Open Phrases, Open Sentences, and Solution Sets

S T A N	C O F S	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
	X	Algebraic vocabulary expansion	1. Use mathematical terminology to translate an equation in various equivalent English sentences. 2. Define and recognize the terms: base, exponent, and power. 3. Determine the degree of monomials and polynomials.			
	X	Open sentences and solution sets	4. Determine the solution set of a given open sentence.			
	X	Translation of worded expressions into algebraic language and their simplification'	5. Translate words into variable expressions and simplify the expressions when necessary. 6. Translate words into equations and simplify the equations when necessary.			
		331	332			

MATHEMATICS: ALGEBRA I

AREA: Postulates and Properties of Operations

S T A N	C O F F	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
	X	Axioms of equality	1. Define the reflexive, symmetric, and transitive properties for all real numbers a, b, and c.			
	X	Closure property	2. Write mathematical expressions demonstrating the reflexive, symmetric, and transitive properties.			
	X	Commutative property	3. Define closure for all real numbers a and b.			
			4. Write a mathematical equation demonstrating the commutative property.			
			5. Identify the commutative property given a mathematical equation expressing the property.			
	X	Associative property	6. Write a mathematical equation demonstrating the associative property.			
			7. Identify the associative property given a mathematical equation expressing the property.			
8		Regrouping of three addends to illustrate the grouping	8. Given an expression, be able to identify another way of writing the expression.			

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MATHEMATICS: ALGEBRA I

AREA: Postulates and Properties of Operations

S T A N	C O F S	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
	X	Identity property	9. Define the identity property for real numbers.			
10			10. Write a mathematical expression demonstrating the identity property of addition and multiplication.			
	X	Distributive property	11. Recognize one as the multiplicative identity.			
8			12. Define the distributive property.			
			13. Name the missing addend in a number sentence illustrating the distribution of multiplication over addition.			
10			14. Recognize an application of the distribution of multiplication over addition.			
8		Multiplicative inverse or axiom of reciprocals	15. Define the multiplicative inverse.			
			16. Recognize the reciprocal of a number.			

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MATHEMATICS: ALGEBRA I

AREA: Operations with Monomials and Polynomials

S T A N	C O F S	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
	X	Addition and subtraction	1. Add and subtract single term algebraic expressions.			
	X	Multiplication	2. Add and subtract multiple term algebraic expressions.			
			3. Multiply single term algebraic expressions.			
			4. Multiply a multiple term algebraic expression by a single term algebraic expression.			
			5. Multiply a multiple term algebraic expression by a multiple term algebraic expression.			
	X	Division	6. Divide single term algebraic expressions.			
			7. Divide a multiple term algebraic expression by a single term algebraic expression.			
			8. Divide a multiple term algebraic expression by a multiple term algebraic expression.			
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MATHEMATICS: ALGEBRA I

AREA: Simple Equations, Inequalities, and Problems

S T A N	C O F S	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
8	X	Solution of simple equations	1. Solve an equation involving addition. 2. Solve an equation involving subtraction. 3. Solve an equation involving multiplication. 4. Solve an equation involving division.			
8,10		Solving of linear equations	5. Solve a linear equation of the form $ax - b = c$. 6. Solve a linear equation of the form $ax - b = c + dx$.			
8,10		Solving of proportions	7. Solve a proportion in which the unknown is a numerator in one ratio and a denominator in the other ratio.			
	X	Writing of word sentences as equations	8. Translate words into expressions and equations using variables. 9. Represent problem situations by expressions and equations using variables. 10. Solve word problems using algebraic equations.			
8,10	*	Evaluation of formulas as equations	11. Evaluate formulas. 12. Evaluate a linear expression.			

MATHEMATICS: ALGEBRA I

AREA: Simple Equations, Inequalities, and Problems

S T A N	C O F S	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
8	*	Solution of inequalities	13. Solve inequalities by substitution. 14. Solve inequalities by transformations. 15. Solve problems involving inequalities. 16. Identify a subset of the solution set for an inequality. 17. Find the solution sets of combined inequalities. 18. Graph the solution sets of inequalities.			

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MATHEMATICS: ALGEBRA I

AREA: Word Problems Using One Variable

S T A N	C O F S	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
	X	Vocabulary development	1. Use vocabulary as it relates to equations.			
			2. Translate words into variable expressions.			
			3. Translate words into variable equations.			
		Analysis of English sentences	4. Represent problem situations by variable expressions.			
8			5. Express a word problem as a number sentence.			
10			6. Identify the expression that can be used to solve a two-step problem.			
8			7. Identify the number sentence appropriate to solving a two-step problem.			
	X	Types of word problems	8. Solve the following types of problems: age, area, cost, digit, fraction, geometry, integers, investment, mixture, money, percent, physics, proportion, puzzle, rate-of-work, ratio, trigonometry, uniform motion, and wage.			
	X	Algebra solutions	9. Use an algebraic method to solve word problems.			

MATHEMATICS: ALGEBRA I

AREA: Special Products and Factoring

S T A N	C O F S	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
	X	Products of two binomials	1. Use a short method to find the product of two binomials.			
	X	Cube of a binomial	2. Determine the product of a binomial to the third power.			
	X	Factoring	3. Factor integers, monomials, polynomials, quadratic trinomials, and trinomial squares.			
	X	Use of factoring to solve second degree equations containing one variable	4. Use factoring to determine the solution sets of quadratic equations.			

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MATHEMATICS: ALGEBRA I

AREA: Operations with Fractions

S T A N	C O F S	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
	X	Review of arithmetic fractions and their expansion to include algebraic fractions	1. Review arithmetic fractions.			
	X	Writing of equivalent algebraic fractions	2. Write equivalent algebraic fractions.			
			3. Simplify algebraic fractions.			
	X	Multiplication and division of algebraic fractions	4. Multiply algebraic fractions.			
			5. Divide algebraic fractions.			
	X	Addition and subtraction of algebraic fractions	6. Express fractions with their least common denominators.			
			7. Add and subtract algebraic fractions.			
	X	Simplification of mixed expressions and complex fractions	8. Write mixed expressions as a fraction in simplest form.			
			9. Simplify complex fractions.			

MATHEMATICS: ALGEBRA I

AREA: Algebraic Sentences Involving Fractions

S T A N	C O F S	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
	X	Solution of equations and inequalities involving fractions	1. Solve fractional equations. 2. Solve fractional inequalities. 3. Graph solution sets of fractional inequalities.			
	X	Solution of word problems	4. Solve word problems involving fractional equations and inequalities.			

MATHEMATICS: ALGEBRA I

AREA: First Degree Equations and Inequalities Containing Two Variables

S T A N	C O F S	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
8		Graphs and functions	1. Locate a point on a grid, given the values of x and y.			
10			2. Name the ordered pair for a point on a grid.			
			3. Locate a point on a coordinate axis, given its coordinates.			
			4. Identify the function that describes a given ordered pair.			
			5. Identify the ordered pair that represents a solution for a graphed function.			
	X	Graphing of a linear equation in two variables	6. Graph linear equations in two variables.			
			7. Graph linear inequalities in two variables.			

S T A N	C O F F S	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
		Graphic and algebraic solutions of systems of linear equations and inequalities (in two variables)	8. Use graphs to solve systems of linear equations in two variables. 9. Graph the solution set of a system of two linear inequalities in two variables. 10. Use the substitution method to solve systems of linear equations in two variables. 11. Use the addition or subtraction method to solve systems of linear equations in two variables. 12. Use multiplication with the addition or subtraction method to solve systems of linear equations. 13. Use systems of linear equations in two variables to solve problems.			

MATHEMATICS: ALGEBRA I

AREA: Real Numbers

S T A N	C O F S	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
	X	Subset of real numbers	1. Define real numbers. 2. Give examples of real numbers. 3. Determine whether a set of given numbers is a subset of the real numbers.			
	X	Subset of irrational numbers	4. Define irrational numbers. 5. Determine whether a set of given numbers is a subset of the irrational numbers.			
	X	Solution of radical (irrational) equations	6. Solve simple radical equations.			

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S T A N	C O S	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
	X	Standard form of quadratic equations	1. Transform quadratic equations into standard form: $ax^2 + bx + c = 0$.			
	X	Graphic and algebraic solutions of a quadratic equation containing one variable	2. Solve quadratic equations by factoring. 3. Solve quadratic equations by completing the square. 4. Solve quadratic equations by using the quadratic formula. 5. Graph quadratic functions.			
	X	Solution of word problems which are translated into quadratic equations	6. Express word problems as quadratic equations. 7. Solve word problems which are translated into quadratic equations.			